Time: 3hrs. M.M. 80

General Instructions:
1. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer type questions of 2 marks each.
4. Section C has 6 Short Answer type questions of 3 marks each.
5. Section D has 4 Long Answer type questions of 5 marks each.
6. Section E has 3 case based studies of 4 marks each.

Section - A

1. Let R be relation in set N given by $R = \{(a, b); a = b - 2, b > 6\}$ then

   a) $(2, 4)\epsilon R$ b) $(3, 8)\epsilon R$ c) $(6, 8)\epsilon R$ d) $(8, 7)\epsilon R$

2. If $f: R \to R$ be defined by $f(x) = \frac{1}{x} \forall x \epsilon R$ then f is

   a) One – one   b) onto   c) bijective   d) f is not defined

3. The domain of the function defined by $f(x) = sin^{-1}\sqrt{x-1}$ is

   a) $[1, 2]$ b) $[-1, 1]$ c) $[0, 1]$ d) none of these

4. The value of $cot\left[\frac{1}{2} sin^{-1}\frac{\sqrt{3}}{2}\right]$

   a) 1 b) $\frac{1}{\sqrt{3}}$ c) $\sqrt{3}$ d) 0

5. The point(s), at which the function 'f' is given by $f(x) = \begin{cases} \frac{x}{|x|}, & x < 0 \\ -1 & x \geq 0 \end{cases}$ is continuous

   a) $x \in R$ b) $x = 0$ c) $x \in R - \{0\}$ d) $x = -1 \ and \ 1$

6. If $y = sin^{-1}x$, then $(1 - x^2)y_2$ is equal to

   a) $-\frac{y}{x}$ b) $\frac{y}{x}$ c) $sec^2\left(\frac{y}{x}\right)$ d) $-sec^2\left(\frac{y}{x}\right)$

7. If $y = -x^2$ value of $\frac{dy}{dx}$ for $x < 0$ is

   a) $2x$ b) $-2x$ c) $\frac{x}{2}$ d) $-\frac{x}{2}$

8. If $sin(cos x) = y$ then value of $\frac{dy}{dx}$ is

   a) $cos(cos x)$ b) $sin x \cos(\cos x)$ c) $sin(cos x)$ d) $-sinx \cos(\cos x)$

A-1

9. The value of x for which $(x - x^2)$ is maximum

   a) $\frac{3}{4}$ b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$

10. The interval in which function $y = x^3 + 6x^2 + 6$ is increasing is

   a) $(-\infty, 4) \cup (0, \infty)$ b) $(-\infty, 4)$ c) $(-4, 0)$ d) $(-\infty, 4) \cup (4, \infty)$

11. The rate of change of the area of a circle with respect to its radius, $r = 6cm$ is

   a) 10π b) 12π c) 8π d) 11π

12. The function $f(x) = \sin x$ is strictly decreasing at interval

   a) $\left(\frac{\pi}{2}, \pi\right)$ b) $(0, \pi)$ c) $\left(-\frac{\pi}{2}, \pi\right)$ d) $(-\pi, \pi)$

13. The total revenue in rupees received from sale of x units of a product is given by
   $R(x) = 3x^2 + 36x + 5$, the marginal revenue when $x = 15$ is

   a) 116   b) 96   c) 90   d) 126

14. The value of 'p' for which the vectors $2i + pj + k$ and $-4x - 6j + 26k$ are perpendicular to each other is

   a) 3   b) -3 c) $-\frac{17}{3}$ d) $\frac{17}{3}$

15. The number of corner points of feasible region determined by the constraints

   $x - y \geq 0,\ 2y \leq x + 2,\ x \geq 0,\ y \geq 0$

   a) 2   b) 3   c) 4   d) 5

16. Given that matrices A and B are of order $3 \times n$ and $m \times 5$ respectively, then order of matrix

   $C = 5A + 3B$ is

   a) 3×5 and $m = n$ b) 3×5 c) 3×3 d) 5×5

17. If $|A| = |KA|$, where A is a square matrix of order 2, the sum of all possible values of k is

   a) 1   b) -1 c) 2   d) 0

18. If C_{ij} denotes the cofactors of element P_{ij} of matrix $P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$ then find value of

   $C_{31} \times C_{23}$

   a) 5   b) 24   c) -1 d) -25

A-2

Assertion – Reason Based Questions

The following questions consists of two statements – Assertion (A) and Reason (R). Answer the question selecting appropriate option given below:

a) Both A and R are true and R is correct explanation for A.
b) Both A and R are true but R is not correct explanation for A.
c) A is true but R is false.
d) A is false but R is true.

19. **Assertion (A):** $|Sin\ x|$ is continuous function.

Reason (R) : If $f(x)$ and $g(x)$ both are continuous functions, then $gof(x)$ is also continuous function.

20. **Assertion (A) :** $f(x) = x^4$ is decreasing in interval $(0, \infty)$

Reason (R) : Any function $y = f(x)$ is decreasing if $\frac{dy}{dx} < 0$

Section – B

21. Prove that function f in surjective where $f: N \rightarrow N$ such that

$$f(x) = \begin{cases} \frac{n+1}{2} & if\ n\ is\ odd \\ \frac{n}{2} & if\ n\ is\ even \end{cases}$$

Is the function injective? Justify your answer.

22. Check if relation R in the set R in the set R of real numbers defined as $R = \{(a, b); a < b\}$ is

   (i)   Symmetric   (ii) transitive

23. Find the value of $tan^{-1}\sqrt{3} - cot^{-1}(-\sqrt{3})$

24. Find λ when the projection of $\vec{a} = \lambda i + j + 4k$ on $\vec{b} = 2i + 6j + 3k$ is 4 units

25. If A and B are symmetric matrices such that AB and BA are both defined then prove that (AB – BA) is a skew-symmetric matrix.

Section – C

26. Prove that $tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \frac{\pi}{4} - \frac{x}{2}, x \in \left(-\frac{\pi}{2}, \pi/2\right)$

27. Find the relationship between $'a'$ and $'b'$ so that function $'f'$ defined by

$$f(x) = \begin{cases} ax + 1 & if\ x \leq 3 \\ bx + 3 & if\ x > 3 \end{cases}$$

Is continuous at $x = 3$

28. If $y = e^{xsin^2x} + (\sin x)^x, find\ \frac{dy}{dx}$

A - 3

29. A particle moves along the curve $6y = x^3 + 2$. Find the points on curve at which the y-coordinate is changing 8 times as fast as x co-ordinate.

30. Prove that $y = \frac{4 \sin \theta}{(2 + \cos \theta)} - \theta$ is increasing function of θ in $(0, \pi/2)$

31. Find the internals in which the function 'f' given by $f(x) = \tan x - 4x, x \in (0, \frac{\pi}{2})$ is

(i)   Strictly increasing   (ii) strictly decreasing

Section – D

32. Show that the relation R is set A, where $A = \{x \in Z; 0 \le x \le 12\}$ given by

$R = \{(a, b); a, b \in Z, |a - b|$ is divisible by $4\}$ is an equivalence relation. Find set of all elements related to 1

33. Prove that the volume of the largest cone that can be inscribed in a sphere of radius 'R' is $\frac{8}{27}$ of the volume of the sphere.

34. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

35. Determine the product

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

and use it to solve the system of equation

$$x - y + z = 4 .$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

Section – E

36. Read the following and answer the questions.

Amit, Biraj and Chirag were given the task of creating a square matrix of order 2. Below are the matrices created by them. A, B , C are the matrices created by Amit, Biraj and Chirag respectively.

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

(i) Find the sum of the matrices A, B and C , $A + (B + C)$.

(ii) Evaluate $(A^T)^T$.

(iii) (a) Find the matrix $AC - BC$.

OR

(iii) (b) Find the matrix of $(a + b)B$ when $a = 4$ and $b = -2$.

A-4

37 **. Read the following passage and answer the following questions.**

Linear Programming Problem is a method of or finding the optimal values (maximum or minimum) of quantities subject to the constraints when relationship is expressed as a linear equations or linear inequations.

The corner points of a feasible region determined by the system of linear constraints are as shown below.
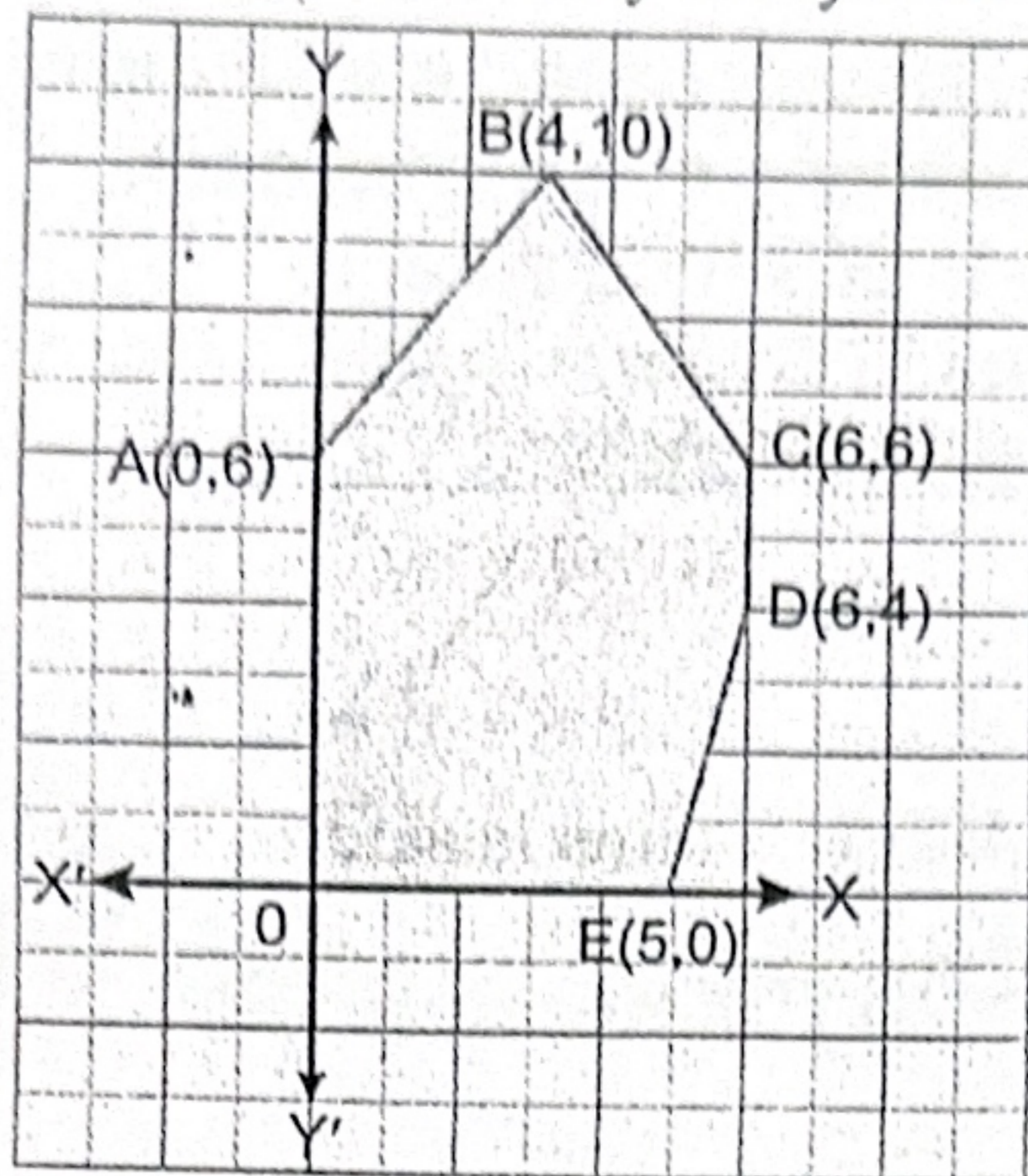

(i) Is this feasible region is bounded?

(ii) Write the number of corner points in the feasible region.

(iii) (a) If $Z = ax + by$ has maximum value at C (6, 6) and B (4, 10). Find the relationship between a & b.

OR

(iii) (b) If $Z = 2x - 5y$ then find the minimum value of this objective function.

38 Read the following passage and answer the following questions.

Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore.

One complete cycle of a four-cylinder four Stroke engine. The volume displaced is marked
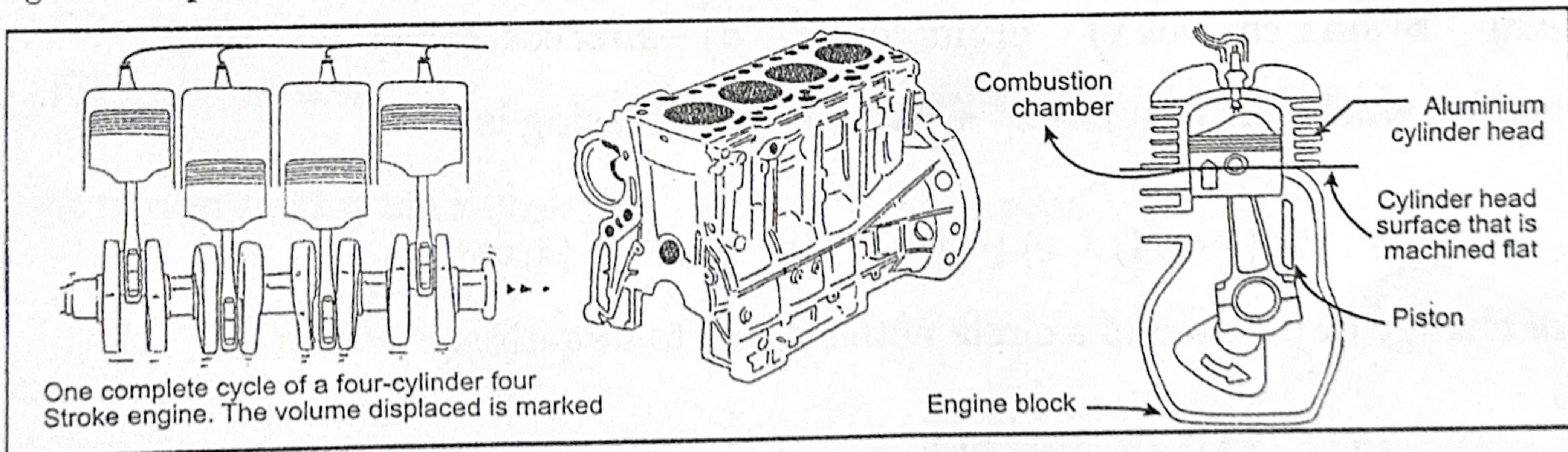
The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r.

(ii) Find $\dfrac{dV}{dr}$.

(iii) (a) Find the radius of cylinder when its volume is maximum.

OR

(iii) (b) For maximum volume, $h > r$. State true or false and justify.

A -5