BUDHA DAL PUBLIC SCHOOL PATIALA

PRE BOARD EXAMINATION (20 January 2024)

Class – XII

Paper-Mathematics (Set-A)

Time: 3hrs. M.M. 80

General Instructions:

1. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer type questions of 2 marks each.
4. Section C has 6 Short Answer type questions of 3 marks each.
5. Section D has 4 Long Answer type questions of 5 marks each.
6. Section E has 3 case based studies of 4 marks each.

Section - A

1. Let $R \to R$ be defined by $(x) = \frac{1}{x}$ $\forall x \in R$. Then

   a) one-one b) on to c) *bijective* d) f is not defined

2. Let Z be the set of integers and R be a relation defined in Z such that aRB if $(a - b)$ is divisible by 5. Then number of equivalence classes are

   a) 2 b) 3 c) 4 d) 5

3. The principle value of $sec^{-1}(-2)$ is

   a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{2\pi}{3}$ d) not defined

4. How many matrices are possible having 24 elements?

   a) 4 b) 6 c) 8 d) 2

5. If area of a triangle with vertices $(k, 0), (1, 1)$ *and* $(0, 3)$ is 5 sq units, then the value(s) of k is

   a) $-\frac{7}{2}$ b) $\frac{13}{2}$ c) $\frac{7}{2}, \frac{-13}{2}$ d) $-\frac{7}{2}, \frac{13}{2}$

6. State which of the following is continuous as well as differentiable for $x \in R$

   a) $|x|$ b) $[x]$ c) polynomial function d) none of theses

7. Derivative of $\frac{x}{x-1}$ with respect to x, is

   a) 2 b) $\frac{1}{(x-1)^2}$ c) $\frac{2x-1}{(x-1)^2}$ d) $\frac{-1}{(x-1)^2}$

8. Given function $f(x) = x^2 e^{-x}$, then $'f'$ increases in the interval

   a) $(-\infty, \infty)$ b) $(-2, 0)$ c) $(2, \infty)$ d) $(0, 2)$

9. Which of the following function is decreasing on $\left(0, \frac{\pi}{2}\right)$?

a) $\cos x$ b) $-\cos 2x$ c) $\cos 3x$ d) $\tan x$

10. The value of $\int_0^2 x[x]dx$ is

a) $\frac{7}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) none of these

11. $\int_1^2 x^2 dx =$

a) 1 b) $\frac{7}{3}$ c) $\frac{1}{3}$ d) 0

12. The area bounded by the curve $x^2 = 4y + 4$ and the line $3x + 4y = 0$ is

a) $\frac{25}{4}$ sq units b) $\frac{125}{8}$ sq units c) $\frac{125}{16}$ sq units d) 31 sq units

13. Area bounded between the parabola $y^2 = 4ax$ and latus rectum is

a) $\frac{4}{3}a^2$ sq units b) $\frac{7}{3}a^2$ sq units c) $\frac{8}{3}a^2$ sq units d) $\frac{8}{3}a$ sq units

14. The integrating factor for the differential equation $\frac{dy}{dx} + y\tan x - \sec x = 0$ is

a) $\tan x$ b) $sec^2 x$ c) $\sec x$ d) $\frac{tan^2 x}{2}$

15. The sum of order and degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = e^x$ is

a) 2 b) 3 c) 5 d) 4

16. The general solution of the differential equation $\frac{dy}{dx} = 2xe^{x^2-y}$ is

a) $e^{x^2-y} = C$ b) $e^{-y} + e^{x^2} = C$ c) $e^y + e^{x^2} = C$ d) $e^{x^2+y} = C$

17. If A and B are two independent events and P (A) = 0.31 and P (B) = 0.41, then $P(A \cap B) =$

a) 0.3141 b) 0.2171 c) 0.1271 d) 0.123

18. If A and B are two independent events and P (A) = 0.4 and P (B) = 0.3, then $P(A \cup B) =$

a) 0.27 b) 0.58 c) 0.6 d) 0.72

In the following questions a statements - Assertion (A) and Reason (R). Answer the question selecting appropriate option given below:

a) Both A and R are true and R is correct explanation for A.
b) Both A and R are true but R is not correct explanation for A.
c) A is true but R is false.
d) A is false but R is true.

19. Assertion (A) : $\begin{bmatrix} -7 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -7 \end{bmatrix}$ is a scalar matrix

Reason (R) : All the elements of the principal diagonal are equal, it is called a scalar matrix.

20. Assertion (A) : The vectors $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 5\hat{k}$ *and* $\vec{b} = 5\hat{\imath} + \hat{\jmath} - 3\hat{k}$ are perpendicular to each other

Reason (R) : $\vec{a} \times \vec{b}$ is a vector perpendicular to both $\vec{a}$ *and* $\vec{b}$

Section – B

21. Evaluate $tan^{-1}\left(tan\frac{9\pi}{8}\right)$

22. If $B = \begin{bmatrix} \propto & 0 \\ 1 & 1 \end{bmatrix}$ *and* $A = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$. Find the value of $\propto$ such that $A = B^2$

OR

If $[2x \quad 3]\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix} = 0$, find the value(s) of x

23. Find the general solution of the differential equation $\frac{dy}{dx} = cos^3x\ sin^4 x$

24. $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, suppose $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and angle between $\vec{b}$ *and* $\vec{c}$ *is* $\frac{\pi}{6}$. Prove that $\vec{a} = \pm 2\ (\vec{b} \times \vec{c})$

25. The probabilities of A, B, C solving a problem, are $\frac{1}{3}, \frac{2}{7}$ *and* $\frac{3}{8}$ respectively. If all the three try to solve the problem of simultaneously, find the probability that exactly one of them can solve the problem.

OR

Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 black balls. One ball is transferred at random from bag I to bag II and then a ball is drawn from bag II. Find the probability that ball drawn is red.

Section – C

26. Show that the function $f = R - \{0\} \to R - \{0\}$ defined by $f(x) = \frac{1}{x}$ is one-one and onto. Is the result true, if the domain $R - \{0\}$ is replaced by N?

27. If $y = Ae^{mx} + Be^{nx}$, Prove that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

28. Differentiate $(\sin x)^x + sin^{-1}\sqrt{x}$

29. Find the interval(s) in which the function f given by $f(x) = \sin x + \cos x, 0 \le x \le 2\pi$, is strictly increasing or strictly decreasing.

30. Evaluate $\int_0^{\frac{\pi}{2}} e^x \sin x \; dx$

31. Find the area of the region bounded by the curves $x = at^2$ *and* $y = 2at$ between the ordinate corresponding to $t = 1$ *and* $t = 2$

OR

Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the ordinates $x = ae$ *and* $x = 0$, where $b^2 = a^2(1-e^2)$ *and* $e < 1$

Section – D

32. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} to solve the system of equations:

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3?$$

OR

For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 4I = O$. Hence find A^{-1}

33. The vector $\vec{b} = 3\hat{\imath} + 4\hat{k}$ is to be written as the sum of two vectors $\vec{\alpha}$ *and* $\vec{\beta}$ where $\vec{\alpha}$ is parallel to $\vec{a} = \hat{\imath} + \hat{\jmath}$ and $\vec{\beta}$ is perpendicular to $\vec{a}$. Find $\vec{\alpha}$ *and* $\vec{\beta}$

34. Solve the following linear programming problem graphically.

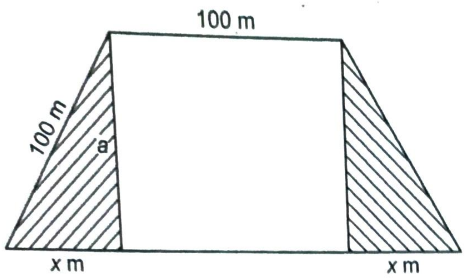
Minimum $Z = 3x + 4y + 370$ subject to the constraints,

$$y \geq 0$$
$$x + y \leq 60$$
$$x \leq 40$$
$$y \leq 40$$
$$x + y \geq 10$$

35. Find the particular solution of the differential equation $dy = \cos x\,(2 - y\, cosec\; x)dx$, given that $y = 2$ then $x = \frac{\pi}{2}$

Section – E

Read the following passage and answer the questions given below.



A resort at the top of a hill decided to, make a relaxation rectangular field with right triangular fields of equal shape and size, for planting flowers, attached to both sides as shown. They also are thinking of maximising the total area. The length of rectangle and hypotenuse of right triangular fields are 100 m each.

(*i*) If base of triangular field is x m. Then find width a of rectangular field.

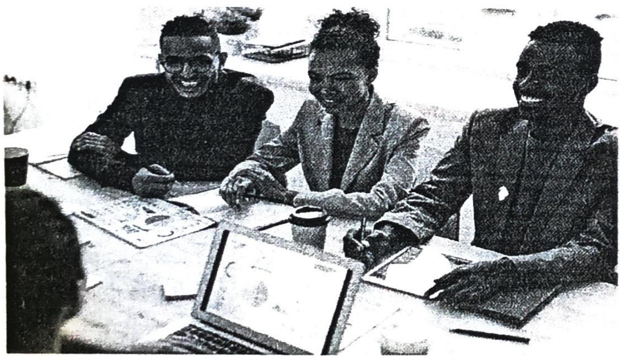
(*ii*) Find the perimeter of total enclosed area.

(*iii*) Find the total covered area.

OR

(*iii*) Find the value of x for which total area is maximum.

37. Read the following passage and answer the questions given below.



In the office three employees Mehul, Janya and Charvi process incoming matter related to a particular project. Mehul processes 40% of the matter and Janya and Charvi process rest of the matter equally. It is found that 6% of matter processed by Mehul has an error where as for Janya and Charvi error rate is 4% and 3% respectively.

(*i*) Find the conditional probability that an error is committed in processing by Janya while processing the matter.

(*ii*) What is the probability that the matter processed by Janya has an error?

(*iii*) What is the probability of an error in processing the matter ?

OR

(*iii*) The processed matter is checked and the selected matter has an error, what is the probability that it was processed by Mehul?

38. **Read the following passage and answer the questions given below.**

A class XII student appearing for a competitive examination was asked to attempt the following questions.

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non zero vectors.

(i) **If $\vec{a}$ and $\vec{b}$ are such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then how $\vec{a}$ and $\vec{b}$ are related?**

(ii) **If $\vec{a} = \hat{i} - 2\hat{j}$, $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ then find $(2\vec{a} + \vec{b}) \cdot [(\vec{a} + \vec{b}) \times (\vec{a} - 2\vec{b})]$.**