CLASS XII

PURE MATHEMATICS(041)

LESSON PLAN: Relation and Functions

No. of days : 15

LEARNING OBJECTIVES:

1. Understand the concept of relations and functions.
2. Types of relations : Reflexive, symmetric , transitive ,equivalence
3. Identify different types of relations and functions: one-one and onto functions
4. Determine the domain, co-domain, and range of a function.
5. Perform operations on functions, such as composition and inverse.
6. Solve problems involving relations and functions

P.K. Testing:

Students will be asked about functions, domain, co-domain and range.

Vocabulary:

One-one function, Equivalence relation, surjective function, bijective function and composite function

Aids used/Resources:

You tube, https://academiceasy.com, smart class, Exam idea.

Interdisciplinary Linkage Activity: To show the interdisciplinary relevance of relations and functions, engage students in the following activity:

1. Discuss real-life applications of relations and functions in various fields, such as computer science, economics, physics, and biology.
2. Invite a guest speaker (e.g., a professional in data analysis or economics) to talk about the practical applications of relations and functions in their field.

Assign a group project where students research and present a case study on how relations and functions are used in a specific interdisciplinary context

Innovative Methods: To make the lesson engaging and interactive, incorporate the following innovative methods:

1. Use visual aids, diagrams, and graphs to illustrate relations and functions.
2. Incorporate technology tools, such as graphing calculators or online function plotter tools, to visualize and explore different functions.
3. Use real-life examples and scenarios to demonstrate the application of relations and functions.
4. Encourage students to collaborate in pairs or small groups to solve problems and analyze functions.

Procedure:

$A = \{1,2,3\}$ $B = \{2,4,5\}$

$A \times B = \{(1,2),\ (1,4),\ (1,5), (2,2),\ (2,4),\ (2,5),\ (3,2), (3,4), (3,5)\}$

Relation in subset of A × B

Equivalence Relation:

i) Reflexive : $\forall a \in A,\ (a,b) \in R$

ii) Symmetric : $if (a,b) \in R \Rightarrow (b,a) \in R$

iii) Transitive : $If\ (a,b) \in R\, and\ (b,c) \in R \Rightarrow (a,c) \in R$

Function:

A relation $f : A \rightarrow B$ becomes function if $\forall$ element of A has one and only one image in set B.

Types of functions:

i) One-one function $\forall\, x_1, x_2 \in X$

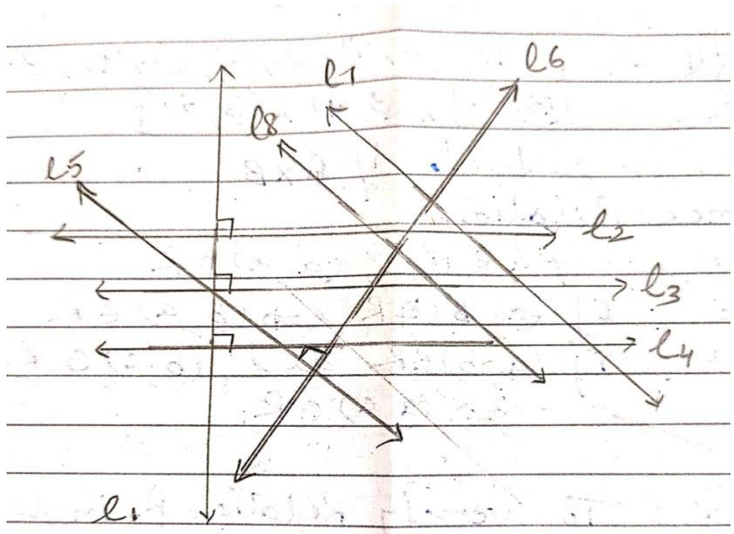
$$if\ \ f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

ii) Onto function $\forall\, y \in Y\ \exists\ x \in X\ such\ that\ \ f(x) = y$

Activity:

To verify Relation R in set L of all lines in plane defined by $R = \{(l,m) : l \perp m\}$ is not equivalence relation.

Take a plywood piece and paste white paper on it. Fix wires with help of nails such that some of them are parallel and some areas to each other and some are inclined as shown in figure :



1. Let the wires represent lines $l_1, l_2, \ldots. l_8$
2. l_1 is perpendicular to $l_2,\ l_3,\ l_4$
3. $l_5 \perp l_6$

$$(l_1, l_2),\ (l_1, l_3),\ (l_1, l_4)\ and\ (l_5, l_6) \in R$$

Remedial Measures on Relations and Functions: for weak , average and gifted students

1. Review Basic Concepts: If students are struggling with the basic concepts of relations and functions, take some time to review the definitions and characteristics of relations and functions. Provide additional examples and explain the differences between the two. Reinforce the concept of domain, co-domain, and range of a function.
2. Practice with Classifying Relations: If students are having difficulty classifying relations (e.g., reflexive, symmetric, transitive), provide them with extra practice problems. Use examples that are relatable and encourage students to think critically and analyze the properties of each relation. Offer guidance and feedback as they work through the problems.
3. Domain and Range Identification: If students are struggling with identifying the domain and range of a function, provide more practice problems that require them to determine the possible input values (domain) and output values (range) of a function. Emphasize the importance of understanding the context and restrictions of the function to correctly determine the domain and range.
4. Function Notation and Graphing: If students are finding it challenging to work with function notation or graph functions accurately, provide step-by-step explanations and examples. Practice writing functions in different forms (e.g., algebraic, graphical, verbal) and guide students in translating between representations. Use visual aids and technology tools to reinforce the connection between function notation and graphing.

5. Individualized Support: If some students require additional support, offer one-on-one or small group sessions to address their specific needs. Provide extra practice materials, offer clarifications, and encourage them to ask questions. Monitor their progress and provide constructive feedback to help them overcome their difficulties.
6. Technology Integration: Incorporate technology tools, such as graphing calculators or online function graphing platforms, to help students visualize and explore functions.

7. Encourage Peer Collaboration: Create opportunities for students to work together in pairs or small groups. Assign collaborative problem-solving tasks or group projects that require students to discuss and apply the concepts of relations and functions. Encourage them to explain their reasoning and learn from each other's approaches.
8. Regular Review and Practice: Schedule regular review sessions to reinforce previously covered concepts. Use revision exercises, quizzes.

Art Integration:

Art integration provides a creative and visual way to explore the concepts of relations and functions. It allows students to express their understanding through artistic representations. Here's an art integration activity for relations and functions:

Activity: Visualizing Relations and Functions

Objective: Create an artistic representation that visually represents the concepts of relations and functions and demonstrates understanding of their properties.

Materials:

- Drawing or painting supplies (paper, pencils, erasers, markers, paints, brushes, etc.)
- Reference materials on relations and functions (textbooks, handouts, online resources)

Instructions:

1. Introduction (10 minutes)
- Review the concepts of relations and functions with the students.
- Explain that they will be creating artwork that visually represents these mathematical concepts.
2. Understanding Relations and Functions (20 minutes)
- Discuss the properties and characteristics of relations and functions.
- Highlight the differences between the two and their relevance in various contexts.
- Show visual examples and representations of relations and functions.
3. Artwork Planning (20 minutes)
- Divide the students into pairs or small groups.
- Provide each group with a large sheet of paper or canvas.
- Instruct the groups to brainstorm and sketch ideas for their artwork, focusing on how they can visually represent relations and functions.
- Encourage creativity and the use of colors, shapes, and symbols to depict the properties and operations of relations and functions.

Assignment Test: (10 marks

Q1. Let the function $f: R \to R$ be defined by $f(x) = 4x - 1, \forall\, x \in R$. Then show that is one – one. (2½)

Q2. Let $A = \{0, 1, 2, 3\}$ and define a relation R on A as follows: (2½)

$R = \{(0,0), (0,1), (0,3), (1,0), (1,1), (2,2), (3,0), (3,3)\}$

Q3. What is the range of the function $f(x) = \frac{|x-1|}{x-1}$? (2½)

Q4. Let $f: R \to R$ be the function defined by $f(x) = x^3 + 5$. Show that $f(x)$ is invertible. Find $f^{-1}(x)$.

Assignment 2:

Q2. Q1. Check whether the relation R in set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is transitive.

Find number of all onto functions from the set $\{1, 2, 3.....n\}$ to itself

Q3. If $f: R \to R$ be defined by $f(x) = \sin x$ and $g: R \to R$ be defined by $g(x) = x^2$ then find fog

Q4. If $f(x) = x^2 + 4$ find $f^{-1}(x)$

(2½)

LESSON PLAN:Matrices

No. of days :20

Objective:

Students will learn about matrices, types of matrices, operation on matrices, transpose of matrix, symmetric, skew symmetric matrices, elementary operation of matrix, inverse of matrices.

P.K. Testing:

Students will be asked about how to arrange numbers in square brackets without repetition.

Vocabulary:

Elements of matrix, row, column, order of matrix

Aids used/Resources:

You tube, https://academiceasy.com, smart class, RD Sharma.

Interdisciplinary Linkage Activity:- To show the interdisciplinary relevance of matrices and determinants, engage students in the following activity:

1. Discuss real-life applications of matrices and determinants, such as computer graphics, economics, physics, and computer science.
2. Invite a guest speaker (e.g., a professional in computer graphics or engineering) to talk about the practical applications of matrices and determinants in their field.
3. Assign a group project where students research and present a case study on how matrices and determinants are used in a specific interdisciplinary context.

Innovative Methods:- To make the lesson engaging and interactive, incorporate the following innovative methods:

1. Use interactive online resources, such as educational websites or simulation tools, to demonstrate matrix operations and properties.
2. Incorporate hands-on activities, such as creating physical matrices using manipulatives (e.g., index cards) and performing operations.
3. Encourage students to collaborate in pairs or small groups to solve matrix-related problems or create their own scenarios to apply matrix concepts.

Art Integration :-

- Divide the students into pairs or small groups.
- Provide each group with a large sheet of paper or canvas.
- Instruct the groups to brainstorm and sketch ideas for their artwork, focusing on how they can visually represent matrices and determinants.
- Encourage creativity and the use of colors, shapes, and patterns to depict matrix concepts.

Procedure:

Square matrix of order 2 $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}_{2\times 2}$

Row and column matrix

Row matrix $\begin{bmatrix} 4 & 5 & 6 \end{bmatrix}_{1\times 3}$

Column matrix $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1}$

Transpose of matrix if $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}_{2\times 3}$

Then $A^T = \begin{bmatrix} 1 & -4 \\ -2 & 2 \\ 3 & 5 \end{bmatrix}_{3\times 2}$

Symmetric Matrix

$\begin{bmatrix} 2 & 1 \\ 1 & 14 \end{bmatrix}$

$aij = aji$

Skew Symmetric Matrix

$\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$aij = aji$

Find inverse of $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ by using elementary row operation

A = I A

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \quad R_2 \to R_2 - 2R_1$$

$$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad R_2 \to R_2 * \left(-\frac{1}{5}\right)$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & -\frac{1}{5} \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix} A$$

$$I = A^{-1}\ A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$$

Find $\begin{vmatrix} 2 & 5 \\ 7 & 9 \end{vmatrix} = 18 - 35 = -17$

Assignment:

Q1. $If \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$ then find matrix A

Q2. $If \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ K & 23 \end{bmatrix}$ then find value of K

Q3. Write square matrix of order 2, which is both symmetrix and skew symmetric

Q4. Show that $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2 - 6x + 17 = 0$. Hence find A^{-1}

: Remedial Measures on Matrices : for weak , average and gifted students

1. Reinforce Conceptual Understanding: If students are struggling with understanding the basic concepts of matrices and determinants, provide additional examples and visual aids to reinforce the concepts. Break down complex ideas into smaller, more manageable parts and provide real-life examples to make the concepts more relatable

2. Practice with Basic Operations: If students are facing difficulties in performing basic operations on matrices, such as addition, subtraction, and scalar multiplication, provide them with extra practice exercises. Offer step-by-step guidance and provide feedback on their work to help them grasp the fundamental operations.
Focus on Matrix Multiplication:

3. Matrix multiplication can be challenging for some students. Offer additional practice problems that gradually increase in complexity. Emphasize the importance of correctly identifying the dimensions of the

matrices involved and show them different strategies for multiplication, such as the row-by-column method

LEARNING OUTCOMES:

Students will learn about type of matrices, transpose of matrices, symmetric and skew symmetric matrices and inverse of matrix.

Assignment Test: (10 marks)

Q1. $If\ A=\begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}, B=\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}\ find\ b_{21}+b_{32}$ (1)

Q2. $If\ A=\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\ B=\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}\ C=\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ Find matrix D such that CD – AB = 0 (1)

Q3. Express $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ as sum of symmetric and skew symmetric matrix and verify (4)

Q4. $If\ A=\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, then show that $A^2=4A-3I$ where I in identify matrix of order 2. Hence find A^{-1}. (4)

LESSON PLAN:DETERMINANTS

No. of days :15

LEARNING OBJECTIVES:

Students will learn about Determinants, properties of determinant, Area of Δ, minors, co-facilors, adjoint and inverse of matrix

P.K. Testing:

Students will be asked about matrices, types of matrices, square matrix.

Vocabulary Used:

Determinant, cofactor, minors, adjoint and inverse of matrices.

Aids used/Resources:

You tube, https://academiceasy.com, smart class, RD Sharma

Art Integration :Art integration can be a powerful tool to explore and understand the concept of determinants. It allows students to visualize and represent determinants through creative and artistic means. Here's an art integration activity for determinants:

Activity: Visualizing Determinants

Objective: Create an artistic representation that visually represents the concept of determinants and demonstrates understanding of their properties and applications.

Materials:

- Drawing or painting supplies (paper, pencils, erasers, markers, paints, brushes, etc.)
- Reference materials on determinants (textbooks, handouts, online resources)

Instructions:

1. Introduction (10 minutes)
- Review the concept of determinants with the students.
- Discuss the importance and applications of determinants in various fields, such as linear algebra, physics, and computer science.
- Explain that they will be creating artwork that visually represents determinants and their properties.
2. Understanding Determinants (20 minutes)
- Explain the definition and properties of determinants, including their role in matrix algebra and their relationship to the invertibility of matrices.
- Show visual examples and representations of determinants, such as matrix layouts and calculations.
3. Artwork Planning (20 minutes)
- Divide the students into pairs or small groups.
- Provide each group with a large sheet of paper or canvas.
- Instruct the groups to brainstorm and sketch ideas for their artwork, focusing on how they can visually represent determinants.

- Encourage creativity and the use of colors, shapes, and symbols to depict the properties and calculations of determinants.

4. Artwork Creation (60 minutes)

- Once the planning is complete, allow the students to start creating their artwork.
- Provide guidance and support as needed, emphasizing the connection between their artwork and the mathematical concepts of determinants.
- Encourage students to incorporate visual elements, such as matrices, arrows, lines, and shapes, to represent determinants and their properties.
- Remind them to use colors and symbols to depict the calculations and transformations involved in determinants.

5. Artwork Presentation and Reflection (20 minutes)

- Ask each group to present their artwork to the class.
- Have the groups explain the elements and symbols they used to represent determinants and their properties.
- Facilitate a class discussion on the different artistic interpretations and connections to the mathematical concepts of determinants.
- Encourage students to reflect on the process and how creating artwork enhanced their understanding of determinants.

Procedure:

1. **Determinant:** Every square matrix can be associated to an expression or a number which is known as its determinant.

Determinant of square matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is given by

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

and determinant of a matrix $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is given by

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

This is known as the expansion of $|A|$ along first row.

In fact, $|A|$ can be expanded along any of its rows or columns.

2. **Singular and Non-singular Matrix:** A square matrix is a singular matrix if its determinant is zero. Otherwise, it is a non-singular matrix.

3. ***(i)*** **Minor:** Let $A = [a_{ij}]$ be a square matrix of order n. Then the minor M_{ij} of a_{ij} in A is the determinant of the sub-matrix of order $(n-1)$ obtained by leaving ith row and jth column of A.

For example, if $A = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \\ 2 & -4 & 3 \end{bmatrix}$, then

$$M_{11} = \begin{vmatrix} 2 & -1 \\ -4 & 3 \end{vmatrix} = 2,\ M_{12} = \begin{vmatrix} -3 & -1 \\ 2 & 3 \end{vmatrix} = -7 \text{ and so on.}$$

(ii) **Cofactor:** The cofactor C_{ij} of a_{ij} in $A = [a_{ij}]_{n\times n}$ is equal to $(-1)^{i+j}$ times M_{ij}.

For example, if $A = \begin{vmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \\ 2 & -4 & 3 \end{vmatrix}$, then

$C_{11} = (-1)^{1+1} M_{11} = M_{11} = 2$ and $C_{12} = (-1)^{1+2} M_{12} = -M_{12} = 7$ and so on

By using the property of determinant, show that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

Sol. LHS $= \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Operating $R_1 \to R_1 + R_2 + R_3$, we get

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Taking $(a+b+c)$ common from first row, we get

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Operating $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get

$$= (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -c-a-b & 0 \\ 2c & 0 & -a-b-c \end{vmatrix}$$

Since determinant of a triangular matrix is equal to product of its diagonal elements

$\therefore \quad = (a+b+c)(a+b+c)(a+b+c) = (a+b+c)^3 =$ RHS

6. If $A = \begin{bmatrix} 8 & 2 \\ 3 & 2 \end{bmatrix}$ then find $|\text{adj } A|$.

Sol. We have $\quad A = \begin{bmatrix} 8 & 2 \\ 3 & 2 \end{bmatrix}$

$\therefore \quad |A| = 16 - 6 = 10$

Now, $\quad |\text{adj } A| = |A|^{2-1} = |10| = 10$

Determine the product $\begin{bmatrix}-4 & 4 & 4\\-7 & 1 & 3\\5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\1 & -2 & -2\\2 & 1 & 3\end{bmatrix}$ and use it to solve the system of equations

$x - y + z = 4,\ x - 2y - 2z = 9,\ 2x + y + 3z = 1.$

ol. $$\begin{bmatrix}-4 & 4 & 4\\-7 & 1 & 3\\5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\1 & -2 & -2\\2 & 1 & 3\end{bmatrix} = \begin{bmatrix}-4+4+8 & 4-8+4 & 4-8+12\\-7+1+6 & 7-2+3 & -7-2+9\\5-3-2 & -5+6-1 & 5+6-3\end{bmatrix}$$

$$= \begin{bmatrix}8 & 0 & 0\\0 & 8 & 0\\0 & 0 & 8\end{bmatrix} = 8\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = 8I$$

$$\Rightarrow \frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\-7 & 1 & 3\\5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\1 & -2 & -2\\2 & 1 & 3\end{bmatrix} = I \qquad \Rightarrow \qquad \begin{bmatrix}1 & -1 & 1\\1 & -2 & -2\\2 & 1 & 3\end{bmatrix}^{-1} = \frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\-7 & 1 & 3\\5 & -3 & -1\end{bmatrix}$$

Given system of equation can be written in matrix form as

$$AX = B \Rightarrow X = A^{-1}B, \qquad \ldots(i)$$

where, $A = \begin{bmatrix}1 & -1 & 1\\1 & -2 & -2\\2 & 1 & 3\end{bmatrix}, X = \begin{bmatrix}x\\y\\z\end{bmatrix}, B = \begin{bmatrix}4\\9\\1\end{bmatrix}$

We have $A^{-1} = \frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\-7 & 1 & 3\\5 & -3 & -1\end{bmatrix}$

Now from (*i*)

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\-7 & 1 & 3\\5 & -3 & -1\end{bmatrix}\begin{bmatrix}4\\9\\1\end{bmatrix} = \frac{1}{8}\begin{bmatrix}-16+36+4\\-28+9+3\\20-27-1\end{bmatrix} = \frac{1}{8}\begin{bmatrix}24\\-16\\-8\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}3\\-2\\-1\end{bmatrix} \qquad \Rightarrow \qquad x = 3, y = -2, z = -1$$

Assignment:

Q1. *Find* $C_{12,\, C_{32}}$ *and* M_{22} *and* M_{12} from matrix $= \begin{bmatrix}1 & 3 & -1\\4 & -5 & 6\\3 & 5 & 2\end{bmatrix}$

Q2. Without expanding by using properties of determinants prove that

$$\begin{vmatrix}x+y & y+z & z+x\\z & x & y\\1 & 1 & 1\end{vmatrix} = 0$$

Q3. Find the area of Δ with vertices A (5,4), B (-2,4) & C (2, – 6)

Q4. Solve the system of linear equations:

$$x+2y+z=4 \quad -x+y+z=0 \ \ and \ \ x-3y+z=2.$$

REMEDIAL MEASURES ON DETERMINANTS: for weak , average and gifted students

1. Understand Properties and Properties of Determinants: If students are struggling with understanding the properties of matrices and determinants, provide clear explanations and examples. Offer opportunities for students to practice applying the properties in different contexts, such as finding determinants using row operations or applying determinant properties to simplify expressions.
2. Work on Problem-Solving Skills: Matrices and determinants are often used to solve real-world problems, such as systems of linear equations or transformations. Provide students with a variety of problem-solving exercises that require them to apply matrix and determinant concepts. Encourage them to think critically and analyze the problem before selecting the appropriate matrix operation.
3. Individualized Support: If some students require additional support, offer one-on-one or small group sessions to address their specific needs. Provide extra practice materials, offer clarifications, and encourage them to ask questions. Monitor their progress and provide constructive feedback to help them overcome their difficulties.
4. Utilize Technology: Incorporate technology tools, such as matrix calculators or online interactive resources, to allow students to visualize and practice matrix and determinant operations. This can help reinforce concepts and provide immediate feedback on their work.
Review and Reinforcement: Regularly review previously covered concepts to ensure that students have a solid foundation. Use revision exercises, quizzes, or games to make the review process interactive and engaging

Learning Outcome:

Students will learn about determinant, inverse of a martrix, adjoint, cofactors, minors.

Assignment Test: (15 marks)

Q1. For what value x, the following matrix is singular? $\begin{vmatrix} 5-x & x+1 \\ 2 & 4 \end{vmatrix}=0$ (1)

Q2. Using properties of determinants, prove that (4)

$$\begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ z-y-z & 2x & 2x \end{vmatrix}=(x+y+z)^3$$

Q3. If $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ then verify $A^3-6A^2+9A-4I=0$ and hence find Assignment on

Matrices and Determinants:

Task 1: Matrix Operations

1. Perform the following matrix operations: a) A = [[2, 3], [4, 1]] and B = [[-1, 2], [3, 0]]. Find A + B and A - B. b) C = [[5, 2, -1], [3, 0, 4]] and D = [[1, -2, 3], [0, 4, -2]]. Find C × D and D × C. c) E = [[1, 2], [3, 4], [5, 6]] and F = [[7, 8], [9, 10], [11, 12]]. Find E × F^T.

Task 2: Determinants

1. Calculate the determinants of the following matrices: a) M = [[4, 2], [3, 1]] b) N = [[-1, 3, 2], [0, 2, -1], [4, 0, 1]] c) P = [[2, -1, 3], [0, 4, 2], [-3, 2, 1]]
2. Solve the following system of equations using Cramer's Rule: 3x + 2y = 7 4x - 3y = 10

Task 3: Inverse of Matrices

1. Find the inverse of the following matrices, if it exists: a) Q = [[2, 3], [1, 5]] b) R = [[1, -2, 3], [0, 4, -1], [2, -1, 1]]
2. Solve the following system of linear equations using matrix inversion: 2x + 3y = 8 5x - 4y = 3

Task 4: Application of Matrices and Determinants

1. Use matrices to solve the following problem: A company sells three types of products: P1, P2, and P3. The sales for each product in three different months are given by the matrix S = [[10, 5, 8], [4, 6, 2], [7, 3, 5]]. Calculate the total sales for each product and the total sales for each month.
2. Apply determinants to solve the following problem:
   Calculate the area of a triangle with vertices (2, 3), (4, -1), and (-3, 6).

Continuity and Differentiability

No. of days : 20

LEARNING OBJECTIVES:

Students will learn about continuity and derivatives of functions, derivatives inverse function, logarithmic functions, exponential function.

P.K. Testing:

Students will be asked about limits derivatives of functions.

Vocabulary:

Continuous, composite functions, chain rule, parametric form, second order derivatives.

Aids used/Resources:

You tube, https://academiceasy.com, smart class, RD Sharma.

Interdisciplinary Linkage Activity (15 minutes):

1. Engage students in an interdisciplinary activity by asking them to find real-life examples where continuity and differentiability are relevant.
2. Divide students into groups and provide them with chart papers and markers.
3. Each group should brainstorm and create a chart illustrating their chosen example, highlighting how continuity and differentiability apply.
4. Allow each group to present their charts to the class and encourage discussions.

Art Integration (15 minutes):

1. Introduce the concept of a continuous function graph and its differentiable points.
2. Provide art supplies (colored pencils, markers, etc.) and plain graph paper to each student.
3. Instruct students to draw graphs of continuous functions with differentiable points.
4. Encourage creativity and emphasize the smoothness of the curves.
5. Allow students to share their artwork with the class, explaining their choices and the significance of differentiable points.

Inclusive Learning Practices (10 minutes):

1. Incorporate inclusive learning practices by promoting peer collaboration and discussion.
2. Assign group activities where students with different abilities work together to solve problems related to continuity and differentiability.
3. Provide additional support to students who may need it by offering extra examples or one-on-one guidance.
4. Use visual aids, diagrams, and real-life examples to cater to diverse learning styles.
5. Encourage students to ask questions and clarify doubts, creating an inclusive and supportive learning environment.

Conclusion and Recap (10 minutes):

1. Summarize the main points covered in the lesson, emphasizing the definitions and conditions for continuity and differentiability.

2. Ask students to share any challenges they faced during the lesson and address any remaining doubts.
3. Provide a brief overview of the upcoming lessons and how continuity and differentiability connect to future topics.
4. Assign homework exercises to reinforce the concepts learned in class.

Procedure:

Continuity of function $f(x)\, at\, x=c\ if\ \underset{x\to c}{lt}\ f(x)=f(c), then\ f(x)\, in\ continuous\, of\ x=c$

$If\ \ f(x)=\begin{cases}3x-4 & 0\le x\le 2\\ 2x+K & 2<x\le 3\end{cases}$ is continuous at $x=2$ find value of K

$\underset{x\to 2}{lt}\ f(x)=\underset{x\to 2}{lt}(2x+K)=3\times 2=4$

$6-4=4+K=6-4$

$\Rightarrow 4+K=2$

$K=-2$

If $f(x)=x^2\sin\frac{1}{x}\ for\ x\ne 0\ iscont\, at\, x=0,\ find\ \ K$

$\underset{x\to 0}{lt}\ f(x)=f(0)=K\Rightarrow \underset{x\to 0}{lt}(x^2\sin\frac{1}{x})=K$

$K=0$

Derivatives:

$\frac{d}{dx}\cos x^3.\sin^2 x$

$\cos x^3\frac{d}{dx}\sin^2 x+\sin^2 x\frac{d}{dx}\cos x^3$

$\cos x^3\times 2\sin x-\cos x\ -3\sin x\sin x^3-x$

$2\sin x\cos x^3.\cos x\ -3\sin x\ \sin x^3-x$

$Find\ \frac{dy}{dx}if\ xy+y^2=\tan x+y$

$$x\frac{dx}{dy}+y.1+2y\frac{dy}{dx}=\sec^2 x+\frac{dy}{dx}$$

$$(x+2y-1)\frac{dy}{dx}=\sec^2 x-y$$

$$\frac{dy}{dx}=\frac{\sec^2 x-y}{x+2y-1}$$

Find derivative of $x^{\sin x}$ $let\ y=x^{\sin x}$ $\log y=\sin x\ \log x$

$$\frac{1}{y}\ \frac{dy}{dx}=\cos x\log x+\sin x\frac{1}{2}$$

$$\frac{dy}{dx}=x^{\sin x}\left[\cos x\ \log x+\frac{\sin x}{x}\right]$$

1. For what value of 'k' is the function $f(x)=\begin{cases}\frac{\sin 5x}{3x}+\cos x, & \text{if } x\neq 0\\ k, & \text{if } x=0\end{cases}$ continuous at $x=0$? [CBSE (F) 2017]

Sol. $\lim_{x\to 0^+} f(x)=\lim_{h\to 0} f(0+h)$

$$=\lim_{h\to 0} f(h)=\lim_{h\to 0}\left(\frac{\sin 5h}{3h}+\cos h\right)$$

$$=\lim_{h\to 0}\frac{\sin 5h}{5h}\times\frac{5}{3}+\lim_{h\to 0}\cos h=1\times\frac{5}{3}+1 \qquad [\because\ h\to 0\Rightarrow 5h\to 0]$$

$\Rightarrow \quad \lim_{x\to 0^+} f(x)=\frac{8}{3}$

Also, $f(0)=k$

Since, $f(x)$ is continuous at $x=0$.

$\Rightarrow \quad \lim_{x\to 0^+} f(x)=f(0) \quad \Rightarrow \quad \frac{8}{3}=k$

<u>LAB ACTIVITY :</u>

OBJECTIVE

To verify Rolle's Theorem.

MATERIALS REQUIRED

A piece of plywood, wires of different lengths, white paper, sketch pen.

APPROACH (METHOD)

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Take two wires of convenient size and fix them on the white paper pasted on the plywood to represent x-axis and y-axis (see Fig. 11.1).
3. Take a piece of wire of 15 cm length and bend it in the shape of a curve and fix it on the plywood as shown in the figure.
4. Take two straight wires of the same length and fix them in such way that they are perpendicular to x-axis at the points A and B and meeting the curve at the points C and D (see Fig. 11.1).

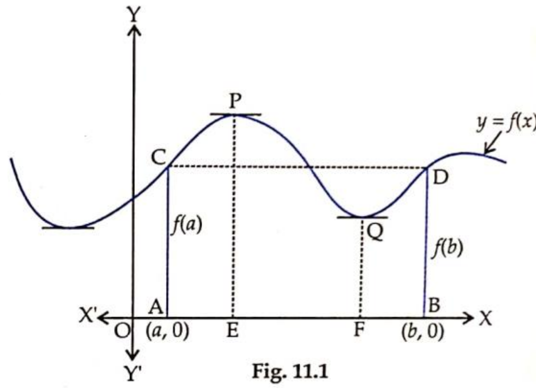


Fig. 11.1

DEMONSTRATION

1. In the figure, let the curve represent the function $y = f(x)$. Let OA = a units and OB = b units.
2. The coordinates of the points A and B are $(a, 0)$ and $(b, 0)$, respectively.
3. There is no break in the curve in the interval $[a, b]$. So, the function f is continuous on $[a, b]$.
4. The curve is smooth between $x = a$ and $x = b$ which means that at each point, a tangent can be drawn which in turn gives that the function f is differentiable in the interval (a, b).
5. As the wires at A and B are of equal lengths, *i.e.*, AC = BD, so $f(a) = f(b)$.
6. In view of steps (3), (4) and (5), conditions of Rolle's theorem are satisfied. From Fig.11.1, we observe that tangents at P as well as Q are parallel to x-axis, therefore, $f'(x)$ at P and also at Q are zero.

   Thus, there exists at least one value c of x in (a, b) such that $f'(c) = 0$.

   Hence, the Rolle's theorem is verified.

Assignment:

Q1. Find the value of a and b if

$$f(x)=\begin{cases} 5 & if\ \ x\le 2 \\ ax+b & if\ \ 2<x<10 \\ 21 & if\ \ x\ge 10 \end{cases}$$

Is continuous function

Q2. If $y = x^{\sin x} + (\sin x)^{\cos x}$ find $\frac{dy}{dx}$

Q3. Find $\frac{dy}{dx}$ if $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

Q4. Verify mean value Theorem if $f(x) = x^2 - 4x - 3$ in interval $[a, b]$ where $a = 1 and\ b = 4$

Remedial Measures for Continuity and Differentiability: for weak , average and gifted students

1. Review Basic Concepts: Start by reviewing the basic concepts of functions, limits, and algebraic operations of functions. Ensure that students have a strong foundation in these areas before moving on to continuity and differentiability.
2. Clear Definitions: Provide clear and concise definitions of continuity and differentiability, along with their significance in calculus. Emphasize the key conditions for a function to be continuous or differentiable.
3. Visual Representations: Utilize visual representations such as graphs and diagrams to help students visualize the concepts of continuity and differentiability. Encourage them to analyze and interpret these visual representations to develop a better understanding.
4. Step-by-Step Problem Solving: Break down problem-solving processes into step-by-step procedures. Guide students through solving problems that involve continuity and differentiability, highlighting the key steps and strategies to follow.
5. Practice Exercises: Provide ample practice exercises with varying difficulty levels to reinforce the concepts of continuity and differentiability. Include exercises that involve applying the conditions for continuity and differentiability, determining points of discontinuity, and finding derivative values.
6. Error Analysis: Review common errors made by students while solving problems related to continuity and differentiability. Identify these errors and discuss them as a class, explaining the correct approach to avoid them in the future.
7. Individualized Assistance: Provide individualized assistance to students who are struggling with the concepts. Offer additional support, explanations, and examples tailored to their specific needs. Provide extra practice materials or worksheets for extra practice if necessary.
8. Peer Collaboration: Encourage peer collaboration and group work during problem-solving activities. Assign students to work in pairs or small groups, allowing them to discuss concepts and problem-solving strategies. This collaborative environment can foster learning and provide opportunities for students to learn from their peers.
9. Concept Reinforcement: Integrate continuity and differentiability into future lessons to reinforce the concepts. Connect these concepts to other topics in calculus, such as limits, derivatives, and applications of derivatives. Show students how continuity and differentiability play a vital role in advanced calculus concepts.
10. Real-World Applications: Illustrate real-world applications of continuity and differentiability to demonstrate their significance outside of mathematics. Show examples from fields such as physics, engineering, economics, and biology, highlighting how these concepts are applied in practical situations.
11. Regular Assessments: Conduct regular assessments, including quizzes, tests, and assignments, to evaluate students' understanding of continuity and differentiability. Provide constructive feedback to help them improve their problem-solving skills and conceptual understanding.
12. Encourage Questions: Create a supportive classroom environment where students feel comfortable asking questions and seeking clarification. Address their doubts promptly and encourage active participation during class discussions.

By implementing these remedial measures, you can provide additional support to students

Learning Outcome:

Students will learn about continuity and derivatives of trigonometric function, second order derivatives, derivatives of logarithmic functions.

Assignment Test: (10 marks)

Q1. $If\ x=\sqrt{a^{\sin-1t}}\ \ y=\sqrt{a^{\cos-1t}}$ show that $\frac{dy}{dx}=-\frac{y}{x}$ (2½)

Q2. For what value of λ is the function defined by $f(x)=\begin{cases}\lambda(x^2-2x) & if\ x\le 0\\ 4x+1 & if\ x>0\end{cases}$ (2½)

Q3. Differentiate $\tan^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\} 0<x<\bar{\wedge}$ (2½)

Q4. Differentiate $(\log x)^x + x^{\log x}$ w.r.t to x (2½)

Inverse Trigonometric Functions

Learning Objectives:

1. Students will be taught about inverse of trigonometric functions. Understand the concept of inverse trigonometric functions and their properties.
2. Apply the properties of inverse trigonometric functions to solve problems.
3. Explore real-life applications of inverse trigonometric functions.

P.K. Testing:

1Give examples of one one and functions
2.draw the graph of sinx , cosx, tanx trigonometric functions?
3.State the domains and ranges of sin(x),cos(x), and tan(x).
4..What is the periodicity of sinx trigonometric function.
5. if sin(x) = 1/2, what is the value in radian?

Vocabulary:

Principal values of trigonometric functions. Domain, range,Trigonometric equations

Aids used/Resources:

You tube, https://academiceasy.com, smart class, RD Sharma.

INTERDISCIPLINARY LINKAGE

a.Discuss real-life applications of inverse trigonometric functions, such as finding angles in navigation, physics, and engineering problems.

b. Give examples of how inverse trigonometric functions are used in calculating distances, heights, and angles in various contexts.

ART INTEGERATION:

1. Trigonometric Art Project: Assign students a project where they have to create artwork that incorporates inverse trigonometric functions. For example, they can create geometric designs using sine and cosine functions or create a visual representation of the periodicity of inverse trigonometric functions. Encourage creativity and allow students to explore different mediums such as drawing, painting, or digital art.
2. Graphing Art: Have students create graphs of inverse trigonometric functions and use them as a basis for creating art. They can plot points on a coordinate plane and connect them to form artistic designs. This activity allows students to visualize the relationship between the graphs of trigonometric functions and their inverses.

3. Trig Sculptures: Ask students to create three-dimensional sculptures that represent the properties or concepts related to inverse trigonometric functions. They can use materials such as clay, wire, or recycled materials to construct their sculptures. This hands-on activity helps students engage with the topic in a tactile and visual manner.
4. Kinetic Art: Explore the connection between inverse trigonometric functions and motion by incorporating kinetic art. Students can design and build interactive sculptures or mobiles that move in patterns related to inverse trigonometric functions. This activity allows them to see the practical application of inverse trigonometric functions in creating dynamic art.
5. Mathematical Art Analysis: Show students examples of artwork that incorporate mathematical concepts, such as fractals or tessellations. Ask them to analyze the artwork and identify any connections to inverse trigonometric functions. This activity encourages critical thinking and helps students recognize the presence of math in art.
6. Collaborative Artwork: Divide the class into groups and assign each group a specific concept or property related to inverse trigonometric functions. Have them create a collaborative artwork that visually represents their assigned concept. This activity promotes teamwork and allows students to share their understanding with their peers.
7. Artistic Mnemonics: Ask students to create artistic mnemonics or visual aids that help them remember the properties or formulas of inverse trigonometric functions. They can create posters, flashcards, or even digital presentations that combine art and text to reinforce their learning.
8.

Procedure:

1 . **INTRODUCTION** (10 minutes) a. Begin the lesson by revising the concept of trigonometric functions (sine, cosine, and tangent) and their graphs. b. Explain that inverse trigonometric functions are used to find angles when the values of trigonometric ratios are given. c. Highlight that inverse trigonometric functions are denoted as sin^(-1(x), cos^(-1)(x), and tan^(-1)(x).

***Definition and Properties (15 minutes)

a. Introduce the inverse trigonometric functions: arcsin(x), arccos(x), and arctan(x). b. Discuss the domains and ranges of each inverse trigonometric function. c. Explain the principal value branch and periodicity of inverse trigonometric functions. d. Emphasize the importance of understanding the ranges of inverse trigonometric functions when solving equations. e. Discuss the properties of inverse trigonometric functions, such as:

- sin(arcsin(x)) = x, arcsin(sin(x)) = x
- cos(arccos(x)) = x, arccos(cos(x)) = x
- tan(arctan(x)) = x, arctan(tan(x)) = x f. Solve simple equations involving inverse trigonometric functions as examples.

2. Examples and Problem Solving (20 minutes) a. Present a few examples that involve finding the values of inverse trigonometric functions. b. Provide step-by-step solutions and explanations for each example. c. Encourage students to attempt solving similar problems on their own. d. Walk around the classroom to offer assistance and clarify doubts.
3. Real-life Applications (10 minutes) a. Discuss real-life applications of inverse trigonometric functions, such as finding angles in navigation, physics, and engineering problems. b. Give examples of how inverse trigonometric functions are used in calculating distances, heights, and angles in various contexts.
4. Summary and Wrap-up (5 minutes) a. Summarize the key concepts covered in the lesson. b. Address any remaining doubts or questions from students. c. Assign practice problems from the textbook or handouts. d. Provide resources for additional practice and self-study, such as online tutorials or videos.

Inverse Trigonometric equation

Find principal value of $Cos^{-1}(-\frac{1}{2})$

$$= \pi - \cos^{-1}(\cos\frac{\pi}{3})$$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Solve

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} \quad x \neq 0$$

$$put\ \ x = \tan\theta$$

$$\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta-1}}{\tan\theta}\right)$$

$$\tan^{-1}(\tan\theta/\tan\theta)$$

$$\tan^{-1}1 = \tan^{-1}(\tan\pi/4)$$

$$= \frac{\pi}{4} \in \left(\pi/2,\ \pi/2\right)$$

<u>Activity: Lab Activity</u>

Objective

To explore the principal value of the function $\sin^{-1}x$ using a unit circle.

Material Required

Cardboard, white chart paper, rails, ruler, adhesive, steel wires and needle.

Procedure

1. Take a cardboard of a convenient size and paste a white chart paper on it.
2. Draw a unit circle with centre O on it.
3. Through the centre of the circle, draw two mutually perpendicular lines X′OX and YOY′ representing x-axis and y-axis, respectively as shown in Fig. 6.1.
4. Mark the points A, C, B and D, where the circle cuts the x-axis and y-axis, respectively as shown in Fig. 6.1.
5. Fix two rails on opposite sides of the cardboard which are parallel to y-axis. Fix one steel wire between the rails such that the wire can be moved parallel to x-axis as shown in Fig. 6.2.
6. Take a needle of unit length. Fix one end of it at the centre of the circle and the other end to move freely along the circle Fig. 6.2.

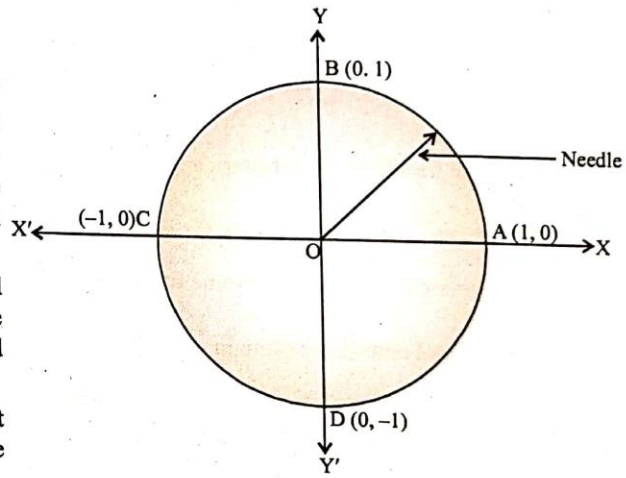


Fig. 6.1

Demonstration

1. Keep the needle at an arbitrary angle, say x_1 with the positive direction of x-axis. Measure of angle in radian is equal to the length of intercepted arc of the unit circle.
2. Slide the steel wire between the rails, parallel to x-axis such that the wire meets with free end of the needle (say at P_1) (Fig. 6.2).
3. Denote the y-coordinate of the point P_1 as y_1, where y_1 is the perpendicular distance of steel wire from the x-axis of the unit circle giving $y_1 = \sin x_1$.
4. Rotate the needle further anticlockwise and keep it at the angle $\pi - x_1$. Find the value of y-coordinate to intersecting point P_2 with the help of sliding steel wire. Value of y-coordinate for the points P_1 and P_2 are same for the different value of angles, $y_1 = \sin x_1$ and $y_1 = \sin(\pi - x_1)$. This demonstrates that sine function is not one-to-one for angles considered in first and second quadrants.

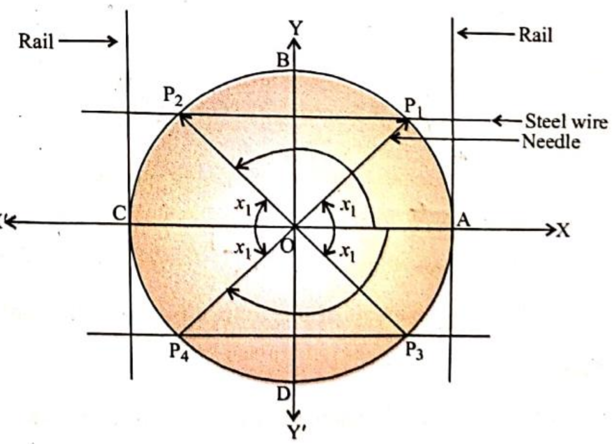


Fig. 6.2

5. Keep the needle at angles $-x_1$ and $(-\pi + x_1)$, respectively. By sliding down the steel wire parallel to x-axis, demonstrate that y-coordinate for the points P_3 and P_4 are the same and thus sine function is not one-to-one for points considered in 3rd and 4th quadrants as shown in Fig. 6.2.

6. But the y-coordinate of the points P_3 and P_1 are different. Move the needle in anticlockwise direction starting from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ and look at the behaviour of y-coordinates of points P_5, P_6, P_7 and P_8 by sliding the steel wire parallel to x-axis accordingly. y-coordinate of points P_5, P_6, P_7 and P_8 are different (See Fig. 6.3). Hence, sine function is one-to-one in the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and its range lies between -1 and 1.

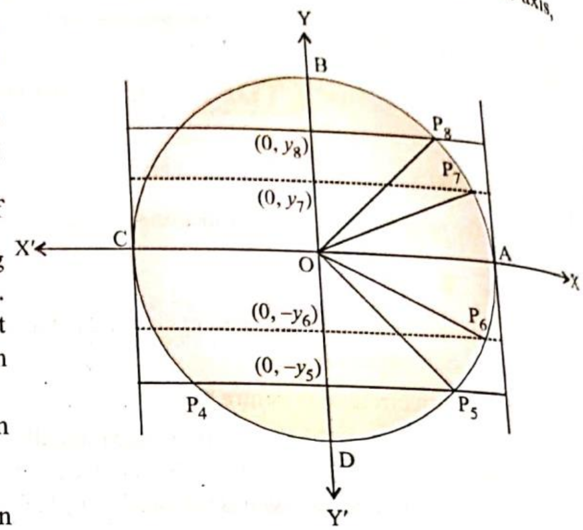


Fig. 6.3

7. Keep the needle at any arbitrary angle say θ lying in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and denote the y-coordinate of the intersecting point P_9 as y. (See Fig. 6.4). Then $y = \sin\theta$ or $\theta = \sin^{-1}y$, as sine function is one-one and onto in the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and range $[-1, 1]$. Therefore, its inverse arc sine function exist. The domain of arc sine function is $[-1, 1]$ and range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. This range is called the principal value of arc sine function (or $\sin^{-1}$ function).

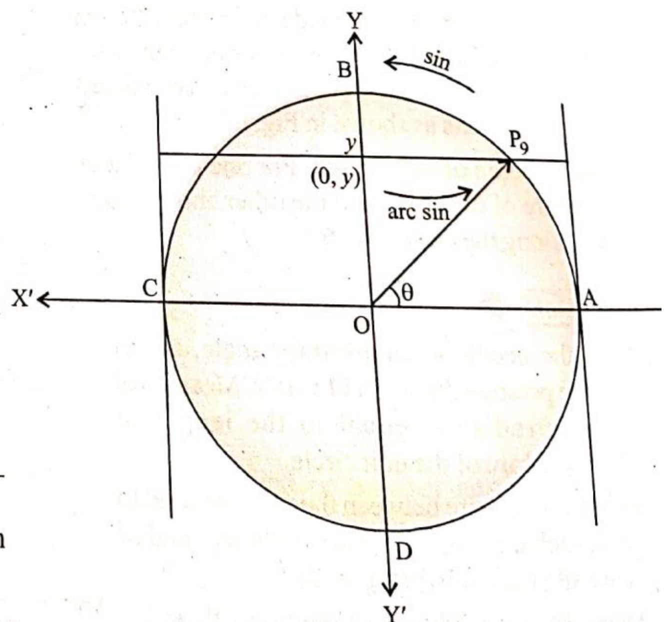


Fig. 6.4

Observation

1. Sine function is non-negative in 1st and 2nd quadrants.
2. For the quadrants 3rd and 4th, sine function is −ve.
3. $\theta = \text{arc} \sin y \Rightarrow y =$ sin θ where $-\frac{\pi}{2} \leq \theta \leq$ π/2.

Assignment:

Q1. Find value of $\cot(\tan^{-1} a + \cot^{-1} a)$

Q2. Find principal value of $\tan^{-1}\left(\tan \frac{-7\pi}{6}\right)$

Q3. If $\sin(\sin^{-1}\frac{1}{5} + \cos^{-1} x) = 1, \textit{ then find value of } x$

Q4. Prove that $\sin^{-1}\left(\frac{63}{65}\right) = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$

Q5. Prove that $\sin^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{5}{12}\right) + \cos^{-1}\left(\frac{63}{65}\right) = \pi/2$

INCLUSIVE LEARNING PRACTICES:

Inclusive practices on inverse trigonometric functions involve creating a supportive and inclusive learning environment where all students feel valued, respected, and able to actively participate in the learning process. Here are some strategies for implementing inclusive practices:

1. Differentiated Instruction: Recognize that students have diverse learning styles, abilities, and needs. Differentiate instruction by offering a variety of instructional methods and materials to accommodate different learning preferences. Provide visual aids, hands-on activities, and auditory explanations to cater to different learning styles.
2. Multiple Representations: Present inverse trigonometric functions using multiple representations, such as graphs, diagrams, tables, and real-life examples. This approach helps students with different learning preferences and strengths to grasp the concepts from different perspectives.
3. Collaborative Learning: Encourage collaboration among students by incorporating group work and peer-to-peer interactions. Assign students to work in pairs or small groups, where they can support each other, discuss concepts, and share their understanding of inverse trigonometric functions. This approach promotes social interaction, cooperation, and mutual learning.
4. Supportive Environment: Foster a positive and inclusive classroom environment by promoting respectful communication, active listening, and valuing diverse perspectives. Create a safe space where students feel comfortable asking questions, sharing their ideas, and making mistakes without fear of judgment. Encourage students to support each other and celebrate each other's successes.
5. Accessibility and Accommodations: Ensure that instructional materials, resources, and assessments are accessible to all students. Consider providing accommodations or modifications for students with specific learning needs or disabilities. This might include providing extra time for assessments, providing visual aids or assistive technology, or offering alternative modes of representation.
6. Multicultural Perspectives: Incorporate diverse cultural perspectives and examples when discussing real-life applications of inverse trigonometric functions. Highlight how different cultures have used trigonometry in various fields such as architecture, astronomy, or traditional art forms. This approach fosters inclusivity and helps students connect mathematics to their own experiences and cultural backgrounds.
7. Formative Assessment and Feedback: Use formative assessment strategies such as questioning, think-pair-share activities, or exit tickets to regularly check students' understanding. Provide timely and constructive feedback that supports their learning and helps them improve their understanding of inverse trigonometric functions.
8. Individualized Support: Offer individualized support to students who may require additional assistance or have specific learning needs. Provide one-on-one guidance, extra practice opportunities, or additional resources to help them grasp the concepts and develop their skills.

9. Sensitivity to Language and Cultural Differences: Be mindful of language barriers or cultural differences that may impact students' understanding. Use clear and concise language, provide explanations in different ways if needed, and be open to different ways of expressing mathematical concepts.

LEARNING OUTCOMES

1. Students would understand concept of inverse trigonometric functions: Students should comprehend the purpose of inverse trigonometric functions, which is to find angles when the values of trigonometric ratios are given.
2. Determine the domains and ranges of inverse trigonometric functions: Students should be able to identify the allowable values (Understand domains) and the corresponding output ranges for each inverse trigonometric function (arcsin(x), arccos(x), arctan(x)).
3. Apply the properties of inverse trigonometric functions: Students should be able to use properties such as sin(arcsin(x)) = x, arcsin(sin(x)) = x, cos(arccos(x)) = x, arccos(cos(x)) = x, tan(arctan(x)) = x, arctan(tan(x)) = x to simplify expressions and solve equations involving inverse trigonometric functions.
4. Solve problems using inverse trigonometric functions: Students should be able to apply inverse trigonometric functions to solve equations and find unknown angles or values in various mathematical and real-life contexts.
5. Interpret and analyze real-life applications: Students should be able to recognize and explain how inverse trigonometric functions are used to solve problems in real-life scenarios, such as navigation, physics, engineering, and geometric calculations.
6. Verify solutions and communicate mathematical reasoning: Students should be able to check the validity of their solutions using inverse trigonometric functions and effectively communicate their mathematical reasoning and steps taken to arrive at the solution.

Assignment Test: (10 marks)

Q1. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\overline{\wedge}}{4}$ (2)

Q2. Prove that $\frac{9\overline{\wedge}}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$ (4)

Q3. Show that $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$ (4)

LESSON PLAN : Application of Derivations

No of days:10

Objective:

Students will be taught about application of Derivatives.

P.K. Testing:

Students already know about derivatives, second order derivatives

Vocabulary:

Rate measure,Increasing functions, decreasing function, tangents, maxima, minima.

Aids used/Resources:

You tube, https://academiceasy.com, smart class, RD Sharma.

Procedure:

Introduction (10 minutes):

1. Begin the lesson by discussing the concept of derivatives briefly. Recap the definition of derivative and its interpretation as the rate of change of a function.
2. Explain to students that derivatives have numerous applications in various fields, such as physics, economics, and engineering.
3. Share a few real-life examples where derivatives play a crucial role, such as finding the maximum profit, determining the rate of change of a population, or optimizing resource allocation.

Rate as measure (30 minutes):

1. Discuss the concept of rate of change in real-life scenarios, such as speed, growth, and decay.
2. Present a problem related to a moving object, such as a car, and ask students to find the rate of change at a specific point.
3. Demonstrate step-by-step how to find the derivative of the function representing the motion and how it provides the rate of change at any given point.
1. Provide additional examples for students to practice finding rates of change in different contexts, such as population growth or chemical reaction .Begin the lesson by revising the concept of derivatives and their interpretation as rates of change.
2. Explain to students that the behavior of a function can be analyzed using derivatives to determine where it increases or decreases.
3. Introduce the terms "increasing function" and "decreasing function" and provide a brief overview of their significance.

Increasing and Decreasing Functions (30 minutes):

1. Define an increasing function as a function whose values increase as the input increases, and a decreasing function as a function whose values decrease as the input increases.
2. Present graphical representations of increasing and decreasing functions and discuss their characteristics.

3. Demonstrate how to determine intervals of increase and decrease by analyzing the sign of the derivative of a function.
4. Provide examples of functions and guide students through the process of finding intervals of increase and decrease.

Local maxima and local minima (40 minutes):

1. Introduce the concept of critical points and explain that they are potential locations for local extrema (maxima or minima) of a function.
2. Explain how to find critical points by setting the derivative of a function equal to zero and solving for the variable.
3. Discuss the first derivative test and its application in determining whether a critical point corresponds to a local maximum or minimum.
4. Demonstrate the process of using the first derivative test to classify critical points and identify local extrema.
5. Provide examples for students to practice finding critical points and classifying them as local maxima or minima.

Objective: The objective of this lesson is to introduce students to the concepts of maxima and minima and equip them with the skills to find maximum and minimum values of functions. By the end of this lesson, students should be able to:

1. Understand the concepts of local and global maxima and minima.
2. Identify critical points and determine their nature using the second derivative test.
3. Apply optimization techniques to solve problems involving maxima and minima.
4. Analyze and interpret real-life scenarios using the concepts of maxima and minima.

1. Begin the lesson by discussing real-life scenarios where we encounter the concepts of maximum and minimum values, such as maximizing profit, minimizing cost, or finding the maximum height of a projectile.
2. Recap the concepts of increasing and decreasing functions and the first derivative test.
3. Introduce the terms "local maximum," "local minimum," and their significance.

Critical Points and maxima n minina points (40 minutes):

1. Define critical points as the points where the derivative of a function is zero or undefined.
2. Explain that critical points can correspond to local extrema or points of inflection.
3. Discuss the second derivative test and its application in determining the nature of critical points.
4. Demonstrate the process of finding critical points, calculating the second derivative, and classifying the critical points as local maxima, local minima, or points of inflection.
5. Provide examples for students to practice finding critical points and classifying them using the second derivative test.

Local maxima and minima and Optimization (40 minutes):

1. Introduce the concept of the maximum and minimum values of a function over its entire domain.
2. Demonstrate step-by-step how to solve optimization problems involving maxima and minima by setting up appropriate equations, finding critical points, and evaluating the function at those points.
3. Provide examples for students to practice solving optimization problems using the concepts of local maxima n minima

INTERDISCIPLINARY LINKAGE :MathemLatics Exploration (40 minutes)

STEP 1

1. students with a set of mathematical problems that involve calculating rates.

2.Examples could include finding the speed of a moving object, calculating the growth rate of a population, or determining the depreciation rate of an asset.

3.Instruct students to work individually or in small groups to solve the problems, using appropriate formulas and units.

1. Discuss the solutions as a class, emphasizing the application of mathematical concepts to calculate rates accurately.

Step 2: Science Data Analysis (40 minutes)

1. Divide students into pairs or small groups.
2. Provide each group with a science data set that exhibits a rate of change (e.g., temperature change over time, enzyme activity, or plant growth).
3. Instruct students to analyze the data and identify the rate of change represented.
4. Ask students to create graphs or charts to visually represent the data and highlight the rate of change.
5. Encourage students to discuss and interpret the implications of the observed rates in scientific terms.

Step 4: Interdisciplinary Reflection (30 minutes)

1. Bring the class together for a discussion on the interdisciplinary nature of rates.
2. Instruct students to reflect on the connections between the mathematical calculations of rates and the scientific interpretation of rates.
3. Ask students to share their observations and insights about how rates are relevant in both mathematics and science.
4. Facilitate a discussion on other disciplines where rates might be important, such as economics, geography, or engineering.

Rate of change of quantities

Find rate of change of area of circle w.r.t. its radius $r, r = 3cm$

$$A = \bar{\wedge} r^2$$

$$\frac{dA}{dr} = 2\bar{\wedge} r, \quad \left(\frac{dA}{dr}\right)_{r=3} = 2\bar{\wedge} \times 3 = 6\bar{\wedge} cm^2 / cm$$

Increasing and decreasing functions

$if\ x_1 < x_2 \Rightarrow f(x_1) = f(x) \rightarrow$ increasing

$if\ x_1 < x_2 \Rightarrow f(x_1) = f(x) \rightarrow$ decreasing

Increasing in $\left[a, b\right]\ if\ \ f(x) > 0 \forall x \in (a,\ b)$

Decreasing in $[a,b]$ $if \quad f(x) < 0 \, \forall x \in (a, b)$

$$f(x) = 2x^2 - 3x$$
$$f^{'}(x) = 4x - 3$$

Increasing $4x - 3 > 0 \quad f(x) > 0$

$$x > \frac{3}{4} \quad x \in \left(\frac{3}{4}, \infty\right)$$

Decreasing $\; f(x) < 0$

$$4x - 3 < 0 \Rightarrow x < \frac{3}{4} x \in (-\infty, \frac{3}{4})$$

Maxima and Minima

Find points of local maxima and minima of function

$$f(x) = x^3 - 6x^2 + 9x - 8$$

$$y_1 = 3x^2 - 12x + 9$$

$$= 3(x^2 - 4x + 3)$$

$$= 3\left[x^2 - 3x - x + 3\right]$$

$$= 3((x-3)\ (x-1))$$

$$y_1 = 0 \Rightarrow x = 3,\ x = 1$$

$x = 1$ is point of local maxima as $\frac{dy}{dx}$ change sign from $+ve\ to -ve$ $x = 3$ is point of local maxima $\frac{dy}{dx}$ change sign from $-ve\ to\ +ve$

Find two numbers whose sum is 24 whose product in as large as possible.

$$x + y = 24 \Rightarrow y = 24 - x \quad p = xy$$

$$p = x(24 - x) = 24x - x^2$$

$$\frac{dp}{dx} = 24 - 2x \Rightarrow \frac{dp}{dx} = 0 \Rightarrow x = 12$$

$$\frac{d^2p}{dx^2} = -2 < 0$$

$\because P$ is maximum when $x = 12$

$x = 12,\ y = 12$

$\therefore$ required number is $12, 12$

Art integration : step1

1. Begin the activity by reviewing the concepts of increasing and decreasing functions.
2. Recap the characteristics and behavior of these functions, emphasizing their graphical representation.
3. Discuss the relationship between mathematics and art, highlighting the ways in which visual representations can help in understanding mathematical concepts.

Step 2: Artistic Representation (60 minutes)

1. Explain to students that they will create artwork inspired by increasing and decreasing functions.
2. Instruct students to choose a specific function or create their own function (linear, quadratic, exponential, etc.).
3. Provide students with the option to use graph paper or printed graph templates as a guide for accuracy, or encourage them to create freehand artwork.
4. Instruct students to visually represent the function on the paper using colors, shapes, patterns, or any other artistic elements.
5. Encourage creativity and artistic expression while ensuring that the artwork captures the essence of an increasing or decreasing function.

Activity: Lab Activity

OBJECTIVE

To understand the concepts of absolute maximum and minimum values of a function in a given closed interval through its graph.

MATERIALS REQUIRED

Drawing board, white chart paper, adhesive, geometry box, pencil and eraser, sketch pens, ruler, calculator.

APPROACH (METHOD)

1. Fix a white chart paper of convenient size on a drawing board using adhesive.
2. Draw two perpendicular lines on the squared paper as the two rectangular axes.
3. Graduate the two axes as shown in Fig. 15.1.
4. Let the given function be $f(x) = (4x^2 - 9)(x^2 - 1)$ in the interval $[-2, 2]$.
5. Taking different values of x in $[-2, 2]$, find the values of $f(x)$ and plot the ordered pairs $(x, f(x))$.
6. Obtain the graph of the function by joining the plotted points by a free hand curve as shown in the figure.

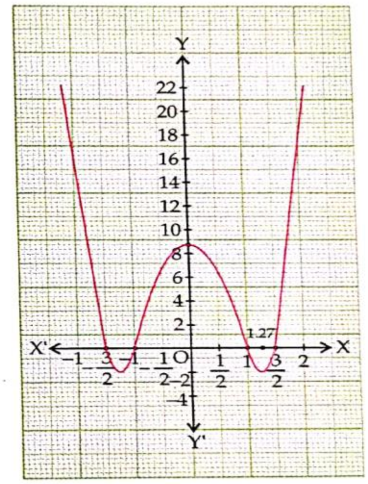


Fig. 15.1

DEMONSTRATION

1. Some ordered pairs satisfying $f(x)$ are as follows:

x	0	±0.5	±1.0	1.25	1.27	±1.5	±2
$f(x)$	9	6	0	–1.55	–1.56	0	21

2. Plotting these points on the chart paper and joining the points by a free hand curve, the curve obtained is shown in the figure.

OBSERVATION

1. The absolute maximum value of $f(x)$ is ....21.......... at x = ...±2........ .
2. Absolute minimum value of $f(x)$ is ....–1.56....... at x = ...1.27....... .

OBJECTIVE

To verify that amongst all the rectangles of the same perimeter, the square has the maximum area.

MATERIALS REQUIRED

Chart paper, paper cutter, scale, pencil, eraser cardboard, glue.

APPROACH (METHOD)

1. Take cardboard of a convenient size and paste a white paper on it.
2. Make rectangles each of perimeter say 48 cm on a chart paper. Rectangles of different dimensions a as follows:

   R_1 : 16 cm × 8 cm, R_2 : 15 cm × 9 cm

   R_3 : 14 cm × 10 cm, R_4 : 13 cm × 11 cm

   R_5 : 12 cm × 12 cm, R_6 : 12.5 cm × 11.5 cm

   R_7 : 10.5 cm × 13.5 cm
3. Cut out these rectangles and paste them on the white paper on the cardboard (see Fig. 18.1 (i) to (

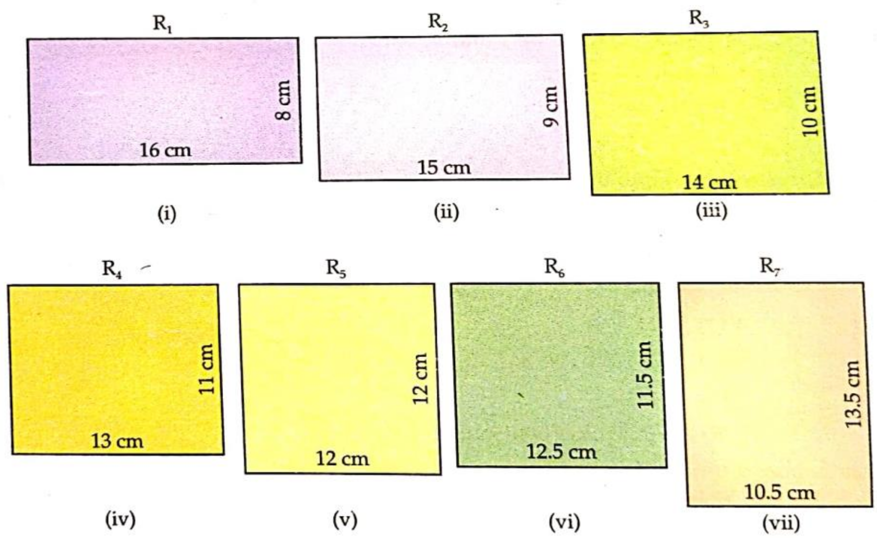


Fig. 18.1

4. Repeat step 2 for more rectangles of different dimensions each having perimeter 48 cm.
5. Paste these rectangles on cardboard.

DEMONSTRATION

1. Area of rectangle of R_1 = 16 cm × 8 cm = 128 cm^2

   Area of rectangle R_2 = 15 cm × 9 cm = 135 cm^2

Area of R_3 = 140 cm^2

Area of R_4 = 143 cm^2

Area of R_5 = 144 cm^2

Area of R_6 = 143.75 cm^2

Area of R_7 = 141.75 cm^2

2. Perimeter of each rectangle is same but their area are different. Area of rectangle R_5 is the maximum. It is a square of side 12 cm. This can be verified using theoretical description given in the note.

OBSERVATION

1. Perimeter of each rectangle R_1, R_2, R_3, R_4, R_5, R_6, R_7 is ....Same.... .
2. Area of the rectangle R_3 ....less.... than the area of rectangle R_5.
3. Area of the rectangle R_6 ....less.... than the area of rectangle R_5.
4. The rectangle R_5 has the diamensions ....12.... × ....12.... and hence it is a ....144 cm^2.... .
5. Of all the rectangles with same perimeter, the ....square.... has the maximum area.

Assignment:

Q1. Find the value of 'a' for which the function $f(x)=\sin x-ax+b$ increasing on R

Q2. If the volume of an air bubble is increasing at rate of 10 cubic feet/min, then find rate of change of its surface area when radius is 1 feet.

Q3 If $y=a\log x+bx^2+x$ has its extreme value at $x=-1 and\ x=2$ then find value of $'a'\&'b'$

Learning Outcome:

Students will learn about how to find maxima, minima, increasing and decreasing function

Assignment Test: (2½ each)

Q1. The length x of rectangle is decreasing at rate of 5cm/min and width y if increasing at rate of 4cm/min. Find rate of change of area of rectangle when $x=8cm\ and\ y=6cm$ (3)

Q2. Find the intervals in which function $f(x)=\sin x+\cos x$, $0\le x\le 2\overline{\wedge}$ is strictly increasing or strictly decreasing. (4)

Q3. Find points on curve $\frac{x^2}{4}+\frac{y^2}{25}=1$ at which the tangents are (4)

(i) parallel to $x-axis$
(ii) parallel to $y\,axis$

Q4. Show that eight circular cone of least curved surface and given volume has an attitude equal to $\sqrt{2}$ times the radius of the base.

LESSON PLAN: INDEFINITE INTEGRALS

No. of days : 20

LEARNING OBJECTIVES

1. Understand the concept of indefinite integrals.
2. Learn the basic properties and rules of indefinite integrals.
3. Apply the techniques of integration to evaluate indefinite integrals.
4. Solve problems involving indefinite integrals.
5. Develop critical thinking skills and problem-solving abilities.

Previous Knowledge Testing: To assess students' prior knowledge, begin the lesson with a short quiz or a class discussion to review the following concepts:

1. Basics of differentiation and the fundamental theorem of calculus.
2. Rules of integration, including the power rule and the constant multiple rule.
3. Applications of integration, such as finding the area under a curve.

Interdisciplinary Linkage Activity: Collaborate with the Physics teacher to demonstrate the application of indefinite integrals in calculating displacement, velocity, and acceleration of a moving object.

AIDS USED: media links, ncert exampler , RD Sharma.

Vocabulary Used:

1. Indefinite integral
2. Anti-derivative
3. Constant of integration
4. Integration by substitution
5. Integration by parts
6. Partial fractions
7. Trigonometric substitution

AIDS USED: You tube media links , ncert exampler ,Rd Sharma

Art Integration: Introduce the concept of integration visually through graphs and geometric shapes. Encourage students to create their own artwork representing different functions and their integrals.

Procedure:

1. Introduction (1 class period): a. Engage students with a real-life example highlighting the need for indefinite integrals. b. Define and explain the concept of an indefinite integral and its relation to differentiation.
2. Present the basic properties and rules of indefinite integrals.

1. $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n, n \neq -1 \Rightarrow \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
2. $\frac{d}{dx}(\log_e x) = \frac{1}{x} \Rightarrow \int \frac{1}{x} dx = \log_e |x| + C$
3. $\frac{d}{dx}(e^x) = e^x \Rightarrow \int e^x dx = e^x + C$
4. $\frac{d}{dx}\left(\frac{a^x}{\log_e a}\right) = a^x, a > 0, a \neq 1 \Rightarrow \int a^x dx = \frac{a^x}{\log_e a} + C$
5. $\frac{d}{dx}(-\cos x) = \sin x \Rightarrow \int \sin x \, dx = -\cos x + C$
6. $\frac{d}{dx}(\sin x) = \cos x \Rightarrow \int \cos x \, dx = \sin x + C$
7. $\frac{d}{dx}(\tan x) = \sec^2 x \Rightarrow \int \sec^2 x \, dx = \tan x + C$
8. $\frac{d}{dx}(-\cot x) = \operatorname{cosec}^2 x \Rightarrow \int \operatorname{cosec}^2 x \, dx = -\cot x + C$
9. $\frac{d}{dx}(\sec x) = \sec x \tan x \Rightarrow \int \sec x \tan x \, dx = \sec x + C$
10. $\frac{d}{dx}(-\operatorname{cosec} x) = \operatorname{cosec} x \cot x \Rightarrow \int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$
11. $\frac{d}{dx}(\log \sin x) = \cot x \Rightarrow \int \cot x \, dx = \log|\sin x| + C$
12. $\frac{d}{dx}(-\log \cos x) = \tan x \Rightarrow \int \tan x \, dx = -\log|\cos x| + C$
13. $\frac{d}{dx}[\log(\sec x + \tan x)] = \sec x \Rightarrow \int \sec x \, dx = \log|\sec x + \tan x| + C$
14. $\frac{d}{dx}[\log(\operatorname{cosec} x - \cot x)] = \operatorname{cosec} x$
$\Rightarrow \int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + C$
15. $\frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right) = \frac{1}{x\sqrt{a^2 - x^2}} \Rightarrow \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
16. $\frac{d}{dx}\cos^{-1}\left(\frac{x}{a}\right) = \int \frac{-1}{\sqrt{a^2 - x^2}} \Rightarrow \int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + C$
17. $\frac{d}{dx}\left(\frac{1}{a}\tan^{-1}\frac{x}{a}\right) = \frac{1}{a^2 + x^2} \Rightarrow \int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
18. $\frac{d}{dx}\left(\frac{1}{a}\cot^{-1}\frac{x}{a}\right) = \frac{-1}{a^2 + x^2} \Rightarrow \int \frac{-1}{a^2 + x^2} dx \frac{1}{a} = \cot^{-1}\left(\frac{x}{a}\right) + C$
19. $\frac{d}{dx}\left(\frac{1}{a}\sec^{-1}\frac{x}{a}\right) = \frac{1}{x\sqrt{x^2 - a^2}} \Rightarrow \int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$
20. $\frac{d}{dx}\left(\frac{1}{a}\operatorname{cosec}^{-1}\frac{x}{a}\right) = \frac{-1}{x\sqrt{x^2 - a^2}}$
$\Rightarrow \int \frac{-1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a}\operatorname{cosec}^{-1}\left(\frac{x}{a}\right) + C$

Techniques of Integration (2 class periods): a. Explore different techniques of integration, such as substitution, integration by parts, partial fractions, and trigonometric substitution. b. Explain each technique with examples, highlighting the steps involved. c. Solve a variety of problems to reinforce the application of integration techniques. d. Provide opportunities for students to practice solving problems independently or in small groups.

3. **Inclusive Practices**: a. Use visual aids and diagrams to aid comprehension. b. Encourage active participation through group discussions and peer learning. c. Provide additional support to students who may need it, such as extra practice problems or one-on-one assistance.

Expected Learning Outcomes: By the end of this lesson, students should be able to:

1. Understand the concept of indefinite integrals and their relationship with differentiation.
2. Apply the basic properties and rules of indefinite integrals effectively.
3. Use various techniques of integration to solve problems involving indefinite integrals.
4. Evaluate integrals involving different functions using appropriate techniques.
5. Demonstrate improved critical thinking skills and problem-solving abilities.

Assignment: Assign a set of problems for students to practice solving indefinite integrals independently. Include a variety of questions that require the use of different integration techniques. Provide a deadline for submission and encourage students to seek help if needed.

Remedial Measures: for weak , average and gifted students

Identify students who require additional support or have difficulty grasping the concepts. Offer remedial classes or one-on-one sessions to address their specific needs. Provide extra practice problems and encourage peer-assisted learning.

LESSON PLAN : Definite integrals

No. of days: 10

Learning Outcomes: By the end of this lesson, students should be able to:

1. Understand the concept of definite integrals and their significance.
2. Apply the fundamental properties of definite integrals.
3. Solve problems involving definite integrals.
4. Relate definite integrals to real-life applications.
5. Use innovative and interdisciplinary methods to enhance learning.
6. Collaborate effectively in groups and engage in inclusive practices.
7. Demonstrate their understanding through an art integration activity.

Previous Knowledge Testing: To assess students' prior knowledge, begin the lesson by giving a short quiz or a set of questions related to indefinite integrals and their properties. This will help gauge their understanding and identify any gaps that need to be addressed during the lesson.

Interdisciplinary Linkage Activity: Incorporate an interdisciplinary activity by connecting definite integrals with physics. Discuss how definite integrals can be used to calculate the area under a velocity-time graph or to find displacement. Engage students in a brief discussion on the applications of definite integrals in physics.

Aidsused/Innovative Method: Media links, ncert exampler. Visualization through Geogebra. Use the Geogebra software to visualize the concept of definite integrals. Demonstrate how the area under a curve can be calculated using definite integrals. Allow students to explore various functions and their corresponding areas using the software. This interactive approach will help students develop a deeper understanding of the topic.

Art Integration: Integrate art into the lesson by asking students to create visual representations of definite integrals. Provide them with colored pencils, markers, or paints, and ask them to draw graphs and shade the areas under the curves to represent definite integrals. This activity will encourage creativity and help students solidify their understanding through a different mode of expression.

Procedure:

1. Warm-up: Begin the lesson by reviewing the concept of indefinite integrals briefly. Ask students to recall the fundamental properties of indefinite integrals.
2. Introduction: Explain the concept of definite integrals and its significance in mathematics and real-life applications. Use real-life examples (e.g., calculating areas, finding volumes) to demonstrate the relevance of definite integrals.
3. Geogebra Visualization: Use Geogebra to visually represent definite integrals. Demonstrate the process of finding the area under a curve and encourage students to explore different functions and their areas.
4. Vocabulary Reinforcement: Introduce and explain relevant vocabulary terms related to definite integrals. Engage students in activities that reinforce their understanding and use of these terms.

PRPPERTIES OF DEFINITE INTEGRALS:

P_0: $\int_a^b f(x)dx = \int_a^b f(t)\,dt$

P_1: $\int_a^b f(x)dx = -\int_b^a f(x)dx$. In particular, $\int_a^a f(x)dx = 0$

P_2: $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

P_3: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

P_4: $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

(Note that P_4 is a particular case of P_3)

P_5: $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

P_6: $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$, if $f(2a-x) = f(x)$ and 0 if $f(2a-x) = -f(x)$

P_7: (i) $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$, if f is an even function, i.e., if $f(-x) = f(x)$.

(ii) $\int_{-a}^a f(x)dx = 0$, if f is an odd function, i.e., if $f(-x) = -f(x)$.

5. Problem Solving: Provide students with a set of problems involving definite integrals. Guide them through step-by-step solutions, encouraging active participation and addressing any difficulties.
6. **Interdisciplinary Linkage**: Connect definite integrals to physics concepts, such as displacement and area under a velocity-time graph. Discuss the applications of definite integrals in physics.
7. **Art Integration Activity**: Assign the art integration activity, where students create visual representations of definite integrals. Provide necessary materials and guidance to support their creative exploration.
8. Assessment: Evaluate students' understanding through formative assessments, such as solving additional problems or participating in a class discussion on the applications of definite integrals.
9. **Assignment**: Assign a set of practice problems to reinforce the concepts covered in the lesson. Provide feedback and address any misconceptions.

Remedial Measures: for weak , average and gifted students

1. Review the Fundamental Theorem of Calculus: Emphasize the connection between differentiation and integration by revisiting the Fundamental Theorem of Calculus. This theorem states that integration and differentiation are inverse operations of each other.
2. Understand the basic integration rules: Make sure students are familiar with the basic integration rules, such as the power rule, constant rule, and sum/difference rule. Reinforce these rules through practice problems.
3. Memorize common integrals: Encourage students to memorize common integrals, such as the integrals of trigonometric functions, exponential functions, logarithmic functions, and some standard functions. This will help them recognize patterns and simplify problem-solving.
4. Practice problem-solving: Provide ample opportunities for students to practice solving integration problems. Start with simple integrals and gradually increase the complexity. Encourage them to solve a variety of problems to develop their problem-solving skills.
5. Use substitution method: Teach students the substitution method, also known as u-substitution, which is a powerful technique for evaluating integrals. Encourage them to identify suitable substitutions and work through examples to gain proficiency.
6. Use integration techniques: Introduce students to various integration techniques, such as integration by parts, partial fractions, trigonometric substitutions, and inverse trigonometric substitutions. Demonstrate when and how to apply each technique through examples.
7. Emphasize algebraic manipulation: Reinforce the importance of algebraic manipulation while solving integration problems. Show students how to simplify expressions, factorize, and expand algebraic terms to make integration more manageable.
8. Provide step-by-step explanations: When teaching integration, break down the steps of the solution process and explain each step in detail. This will help students understand the reasoning behind each technique and foster a deeper comprehension.

9. Encourage graphing: Illustrate the graphical interpretation of integrals. Encourage students to sketch graphs of functions and understand how the area under the curve relates to the integral. This visual representation can aid in understanding and problem-solving.
10. Seek additional resources: Encourage students to explore additional resources like textbooks, online tutorials, video lectures, and practice problem sets to reinforce their understanding of indefinite integrals.

Remember, practice and perseverance are key to improving integration skills. Encourage students to actively engage in solving integration problems and seek help when needed.

Topic: Applications of Integrals

No. of days: 10

Learning Outcomes:

- Understand the concept of finding the area under curves using integration.
- Apply integration techniques to calculate areas of different shapes.
- Relate the concept of area under curves to real-life applications.

Previous Knowledge Testing: To assess the students' prior knowledge, begin the class with a quick review of integration concepts covered in previous lessons. Ask questions to test their understanding of basic integration techniques and concepts such as the definite integral, anti-derivatives, and the fundamental theorem of calculus.

Interdisciplinary Linkage Activity: To establish a connection between mathematics and real-life applications, engage students in an interdisciplinary activity. Ask them to research and present examples of how integrals are used in other subjects like physics, economics, engineering, or computer science. This activity will help students appreciate the practical relevance of integration.

Aids used/Innovative Method: Visualization and Real-life Examples To make the topic more engaging, use visual aids such as graphs, diagrams, and real-life examples during the lesson. Introduce practical scenarios where integration is applied, such as calculating the area between curves or finding the volume of a solid of revolution. Encourage students to visualize the problem and relate it to real-world situations.

Vocabulary used: Introduce and reinforce relevant vocabulary related to the topic, such as integrand, limits of integration, area under the curve, volume of revolution, etc. Provide clear definitions and examples to ensure students understand the terminology used in the lesson. Encourage students to use this vocabulary while discussing and solving problems.

Art Integration: Incorporate art integration by asking students to create visual representations of different functions and their integrals. This can be done through drawing, painting, or using digital tools. This activity will help students develop a deeper understanding of the relationship between a function and its integral.

1. **Procedure**:
2. *Introduction* (15 minutes)

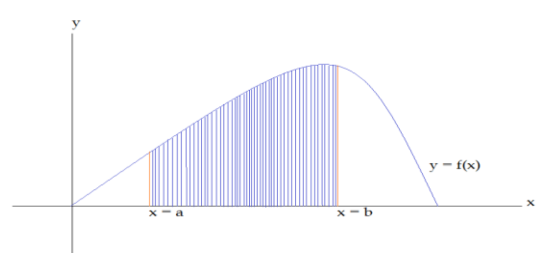
- Begin the lesson by discussing the concept of area and its importance in various fields.
- Introduce the idea of finding the area under curves using integration.
- Explain how integration provides a mathematical tool to calculate these areas accurately.

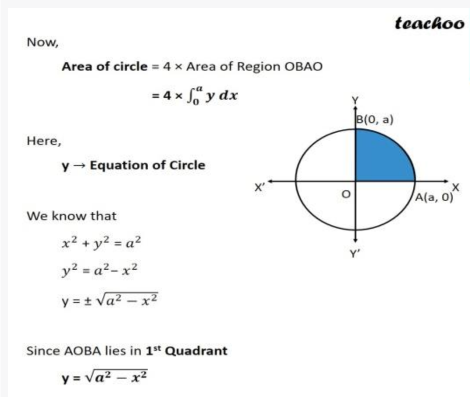
3. Review of Integration Basics (20 minutes)

- Briefly review the fundamental concepts of integration, including the definite integral and anti-derivatives.
- Recap the integration rules, such as the power rule, constant rule, and sum/difference rule.

4. Finding the Area Under a Curve (30 minutes)

- Explain the concept of dividing the region under a curve into small partitions or rectangles ,these partitions approaches zero to form the integrals.





Demonstrate how to set up the integral to calculate the area under a curve, emphasizing the use of appropriate limits of integration.

5. Area of Simple Shapes (40 minutes)

- Introduce simple geometric shapes, such as rectangles, triangles, and trapezoids.
- Derive the formulas for calculating the area of these shapes using integration.
- Guide students through examples and practice problems to reinforce the concept.

6. Area Between Curves (30 minutes)

- Discuss situations where the area between two curves needs to be determined.
- Explain how to set up the integral and choose the appropriate limits of integration for calculating the area between curves.
- Provide examples and practice problems for students to solve.

7. Real-life Applications (20 minutes)

- Present real-life applications where finding the area under curves is relevant, such as calculating volumes of irregular shapes or evaluating probability densities.
- Discuss the significance of accurately calculating areas in these applications.
- Engage students in a class discussion to brainstorm additional real-life scenarios where finding the area under curves is useful.

8. Recap and Practice (15 minutes)

- Summarize the key concepts covered in the lesson.
- Encourage students to ask questions and clarify any doubts.
- Assign practice problems for homework to reinforce understanding.

9. Assessment (Ongoing)

- Assess students' understanding through formative assessments, such as class participation, problem-solving exercises, and quizzes.
- Provide constructive feedback and additional support as needed.

INCLUSIVE PRACTICES : Provide multiple examples and practice problems to cater to different learning styles. Encourage peer collaboration and group discussions to foster a supportive learning environment. Use inclusive language and ensure that all students feel comfortable asking questions. Provide extra assistance to students who need additional support.

Expected Learning Outcomes: By the end of the lesson, students should be able to:

1. Apply integration techniques to solve problems related to area, volume, and other practical applications.
2. Demonstrate a clear understanding of the vocabulary associated with applications of integrals.
3. Apply critical thinking and problem-solving skills to solve complex integration problems

REMEDIAL MEASURES :

Review the Basics: Ensure that students have a strong understanding of the fundamental concepts required for calculating areas, such as integration, definite integrals, and Ani-derivative.

If necessary, provide a quick recap or revise the essential concepts before proceeding.

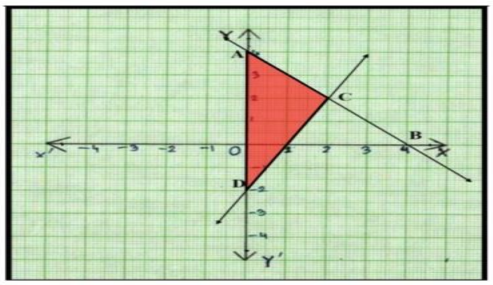
1. ***Geometric Interpretation***: Emphasize the geometric interpretation of finding areas under curves. Help students visualize the regions bounded by curves and the x-axis, and explain how integration enables us to calculate their areas accurately.
2. ***Identify the Curves***: Train students to identify and understand the equations of curves involved in the problem. Discuss different types of curves they may encounter, such as polynomials, exponential functions, trigonometric functions, or combinations of these. Encourage them to sketch the curves to aid in visualization.
3. Determine Limits of Integration: Teach students how to identify the appropriate limits of integration for a given problem. Emphasize the importance of understanding the range over which the area needs to be calculated. Encourage them to carefully analyze the given interval or any additional information provided.
4. Integration Techniques: Reinforce various integration techniques, such as substitution, integration by parts, or partial fractions. Help students choose the most appropriate technique for each problem based on the given curve and simplify the integrals.
5. Practice with Simple Shapes: Begin with simple geometric shapes, such as rectangles, triangles, or trapezoids. Guide students in deriving the formulas for calculating their areas and relate these formulas to integration. Encourage them to apply these formulas to find the areas under simple curves.
6. **ASSIGNMENT :**

1. Find the area of the region bounded by the curves $y^2 = 9x$, $y = 3x$.
2. Find the area of the region bounded by the parabola $y^2 = 2px$, $x^2 = 2py$.
3. Find the area of the region bounded by the curve $y = x^3$ and $y = x + 6$ and $x = 0$.
4. Find the area of the region bounded by the curve $y^2 = 4x$, $x^2 = 4y$.
5. Find the area of the region included between $y^2 = 9x$ and $y = x$
6. Find the area of the region enclosed by the parabola $x^2 = y$ and the line $y = x + 2$
7. Find the area of region bounded by the line $x = 2$ and the parabola $y^2 = 8x$
8. Sketch the region $\{(x, 0) : y = \sqrt{4 - x^2}\}$ and x-axis. Find the area of the region using integration.
9. Calcualte the area under the curve $y = 2\sqrt{x}$ included between the lines $x = 0$ and $x = 1$.
10. Using integration, find the area of the region bounded by the line $2y = 5x + 7$, x-axis and the lines $x = 2$ and $x = 8$.
11. Draw a rough sketch of the curve $y = \sqrt{x-1}$ in the interval [1, 5]. Find the area under the curve and between the lines $x = 1$ and $x = 5$.
12. Determine the area under the curve $y = \sqrt{a^2 - x^2}$ included between the lines $x = 0$ and $x = a$.
13. Find the area of the region bounded by $y = \sqrt{x}$ and $y = x$.
14. Find the area enclosed by the curve $y = -x^2$ and the straight lilne $x + y + 2 = 0$.
15. Find the area bounded by the curve $y = \sqrt{x}$, $x = 2y + 3$ in the first quadrant and x-axis.

7. MCQ:

Question 22 teachoo

Given below is the graph representing two linear equations by lines AB and CD respectively. What is the area of the triangle formed by these two lines and the line x = 0?



(a) 3sq. units (b) 4sq. units (c) 6sq. units (d) 8sq. units

Lesson Plan :Differential Equations

No. of days : 15

Learning Objectives:

1. Understand the concept of differential equations and their applications.
2. Differentiate between ordinary and partial differential equations.
3. Solve first-order linear and separable differential equations.
4. Solve homogeneous and non-homogeneous second-order linear differential equations.
5. Apply differential equations to real-life problems, such as population growth, exponential decay, and mixing problems.
6. Recognize the importance of initial conditions and boundary conditions in solving differential equations.
7. Understand the concepts of general solutions and particular solutions.
8. Use technology tools, such as graphing calculators or software, to solve and visualize differential equations.

Previous Knowledge Testing: Conduct a brief pre-assessment to gauge students' prior knowledge of calculus and algebraic concepts related to derivatives and integrals. Ask questions to assess their understanding of functions, derivatives, and integrals. This will help identify any knowledge gaps and provide a baseline for the lesson.

Interdisciplinary Linkage Activity: Organize a collaborative activity with the physics or biology department. Students can explore how differential equations are used to model physical systems or biological processes. For example, they can work on a project that involves modeling the motion of a falling object or studying population growth using differential equations. This activity will highlight the interdisciplinary nature of differential equations and demonstrate their practical applications.

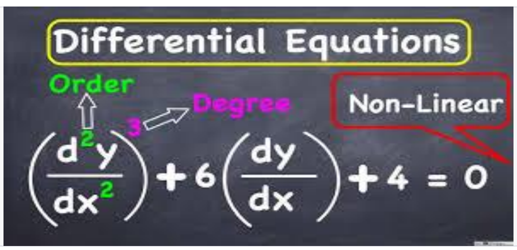
Aids used/Innovative Methods: media links , ncert exampler

1. Interactive Demonstrations: Use online interactive simulations or dynamic software to demonstrate the behavior of differential equations. For example, show how changing the initial conditions or parameters affects the solution of a differential equation.
2. Problem-Based Learning: Present real-life problems that can be modeled using differential equations and guide students through the problem-solving process. Encourage them to analyze the problem, identify the appropriate differential equation, and solve it to find a solution that fits the given conditions.

Vocabulary Used: Differential equation, Ordinary differential equation, Partial differential equation, Linear differential equation, Separable differential equation, Homogeneous differential equation, Non-homogeneous differential equation, General solution, Particular solution, Initial conditions, Boundary conditions.

Procedure:

1. Introduction (10 minutes):
- Engage students by discussing the importance of differential equations in various fields, such as physics, engineering, and economics.
- Introduce the learning objectives for the lesson.
2. Concept Explanation (20 minutes):
- Define differential equations and formulating differential equation by removing constants.
- Explain the basic terminology and vocabulary associated with differential equations.
- Present examples and applications of differential equations in real-life scenarios.



1. Solving First-Order Differential Equations (30 minutes):
- Review the concept of separable differential equations and guide students through solving examples step-by-step.
- Explain the technique of solving linear first-order differential equations using integrating factors.
-

Integrating Factor Method

- $\frac{dy}{dx} + P(x)y = Q(x)$
- $I.F = e^{\int P(x)dx}$

-

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$y^2\,dx + (x^2 - xy - y^2)\,dy = 0$

$y^2\,dx = (x^2 - xy - y^2)dy$

$\frac{dy}{dx} = \frac{y^2}{x^2 - xy - y^2}$

Let $F(x, y) = \frac{dy}{dx} = \frac{y^2}{x^2 - xy - y^2}$

Finding $F(\lambda x, \lambda y)$

$F(\lambda x, \lambda y) = \frac{-\lambda^2 y^2}{\lambda^2 (x^2 - xy - y^2)} = \frac{y^2}{x^2 - xy - y^2}$

$= \lambda^0 F(x, y)$

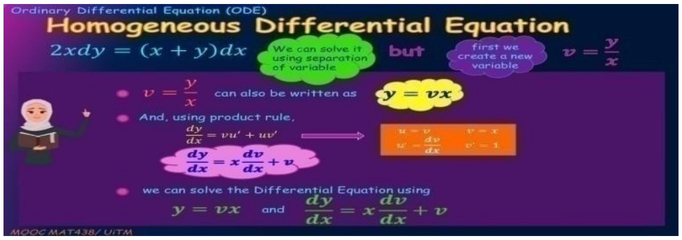
F (x, y) is *a* homogenous function of degree zero.

Solving a First Order Linear Differential Equation:

1. Put the equation into standard form: $\frac{dy}{dx} + P(x)y = Q(x)$
2. Identify P(x) and Q(x).
3. Find: $\int P(x)dx$
4. Let $u(x) = e^{\int P(x)dx}$
5. Solve for y: $y\,u(x) = \int Q(x)u(x)dx + c$

* Discuss homogeneous and non-homogeneous differential equations and provide examples.

- Demonstrate the method of solving these equations using characteristic equations and variation of parameters.



Given equation, $\dfrac{dy}{dx} = \dfrac{x+y}{x-y}$

It is a homogeneous equation so putting $y = vx$

and $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$, we get

$$v + x\frac{dv}{dx} = \frac{x+vx}{x-vx} = \frac{1+v}{1-v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{1-v}$$

$$\Rightarrow \frac{1}{x}dx = \left(\frac{1}{1+v^2} - \frac{v}{1+v^2}\right)dv$$

$$\Rightarrow \log_e x = \tan^{-1} v - \frac{1}{2}\log(1+v^2) + \log_e c$$

Substituting $v = \dfrac{y}{x}$, we get

$$\log_e x = \tan^{-1}\frac{y}{x} - \frac{1}{2}\log\left[1 + \left(\frac{y}{x}\right)^2\right] + \log_e c$$

$$\Rightarrow c(x^2+y^2)^{1/2} = e^{\tan^{-1}(y/x)}.$$

1. **Application and Real-Life Examples** (15 minutes):

- Present real-life problems that can be modeled using differential equations.
- Guide students in setting up the differential equations and solving them to find solutions that fit the given conditions.
- Discuss the interpretation of the solutions in the context of the problems.

2. **Inclusive Learning Strategies** (15 minutes):

- Encourage class participation by asking questions, providing think-pair-share opportunities, and facilitating group discussions.
- Provide visual aids, diagrams, and graphs to support visual learners.
- Incorporate hands-on activities or interactive simulations for kinesthetic learners.
- Offer additional support and resources for students with diverse learning needs.

3. **Expected Learning Outcomes** (5 minutes):

- Summarize the key concepts covered in the lesson.

*Discuss the expected learning outcomes,
such as the ability to solve first-order and homogeneous differential equations and apply them to real-life problems.
*Reviewing Prerequisite Concepts:
*Start by revisiting the foundational concepts required for differential equations, such as calculus, algebra, and functions.

Remedial measures : for weak , average , gifted students
1. Address any knowledge gaps or misconceptions that may hinder their understanding of differential equations.
2 .Providing Additional Practice: Offer extra practice problems and exercises that gradually increase in difficulty. Include a mix of solved examples and challenging problems to help students build their problem-solving skills and confidence.
3 .Individualized Support: Identify specific areas where students are struggling and provide personalized assistance. Offer one-on-one guidance, answer questions, and provide additional explanations as needed. Tailor your support to meet each student's needs.
4. Step-by-Step Approach: Teach students a systematic approach to solving differential equations, emphasizing the step-by-step process. Encourage them to analyze the problem, identify the appropriate method or technique, and carefully follow the steps to arrive at a solution.

5. Visualization Techniques: Incorporate visual aids, such as graphs and diagrams, to help students understand the behavior of solutions to differential equations. Use technology tools, such as graphing calculators or software, to visualize and analyze solutions.
6. Real-Life Applications: Connect differential equations to real-life applications to enhance students' engagement and motivation. Illustrate how differential equations are used to model and solve problems in various fields, such as physics, engineering, and biology.
7. Group Work and Peer Learning: Encourage students to work collaboratively in groups or pairs.

Assignment (10 minutes):

- Assign practice problems that reinforce the concepts covered in the lesson.
- Include a mix of problems involving first-order and second-order differential equations, as well as applications in various contexts.

Lesson Plan: VECTORS

No of days: 15

Learning Objectives: Understand the concept of vectors and their properties.

1. Learn how to represent vectors graphically and algebraically.
2. Apply vector operations, including addition, subtraction, scalar multiplication, and dot product.
3. Solve vector problems in real-life contexts.

4. Develop critical thinking and problem-solving skills through vector analysis.

Previous Knowledge Testing: To assess students' prior knowledge, begin the lesson with a short quiz or review questions related to basic algebra, Encourage students to make connections between the physics principles learned and their application in engineering and construction.coordinate geometry, and trigonometry concepts.

Vocabulary Used:

1. Vector
2. Magnitude
3. Direction
4. Scalar
5. Component
6. Resultant
7. Collinear
8. Coplanar
9. Position vector
10. Displacement

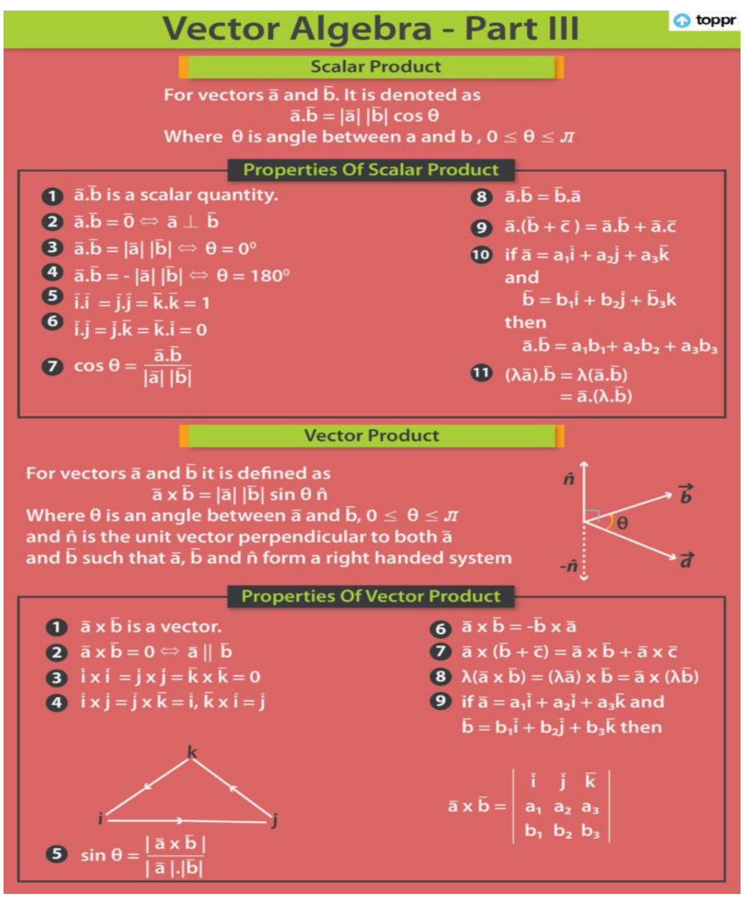
AIDS USED: media links, smart board, ncert ,Rd sharma
Interdisciplinary linkage activity:
*Discuss how vector quantities, such as forces, can be represented and manipulated to ensure the stability and strength of the bridge.
*Encourage students to make connections between the physics principles learned and their application in engineering and construction.
*Explain that the bridges should incorporate vector principles to distribute forces effectively.
ART INTEGRATION(10 minutes):

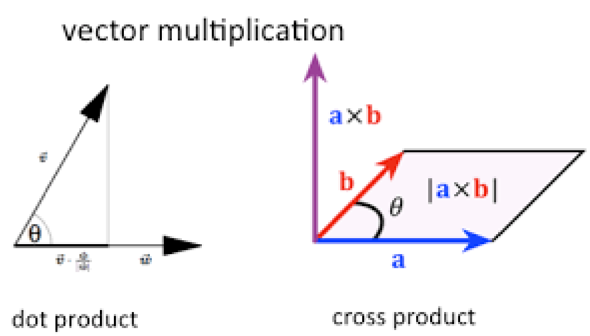
- Demonstrate how to create a simple vector-based artwork step by step.
- Start with a blank sheet of paper and discuss how vectors can be used to create lines and shapes.
- Begin with a vector representing magnitude and direction, such as an arrow.
- Show how vector addition and subtraction can be used to create more complex shapes or compositions.
- Explain how scalar multiplication can be applied to alter the size or intensity of the vectors used in the artwork.

Procedure:

1. Introduction (5 minutes) a. Begin by explaining the definition and characteristics of vectors.
2. b. Highlight the difference between vectors and scalars.

   c. Discuss the importance and applications of vectors in various fields.
3. Vector Representation (10 minutes)
4. a. Explain different ways to represent vectors: geometrically using arrows, algebraically using column matrices, and in terms of their components.
5. b. Demonstrate how to find the magnitude and direction of a vector.
6. Vector Operations (15 minutes)
7. a. Introduce vector addition and subtraction using graphical and algebraic methods.
8. b. Discuss scalar multiplication and its impact on the magnitude and direction of vectors.
9. c. Teach the concept of dot product and Vector product, its geometric interpretation.
10. d. Provide examples and practice exercises to reinforce understanding.





Inclusive Learning (10 minutes) a. Incorporate visual aids, diagrams, and color-coding to support visual learners. b. Encourage group work and peer discussions to promote collaboration and diverse perspectives.
c. Provide extra guidance and support for students with learning difficulties, if necessary.

Expected learning outcomes: students would be able to

1.Define and differentiate between vectors and scalars.

- Comprehend the properties of vectors, including magnitude and direction.
- Explain vector representation using geometric, algebraic, and component methods.

2. Proficiency in Vector Operations:

- Perform vector addition and subtraction using graphical and algebraic methods.
- Apply scalar multiplication to vectors and analyze its impact on magnitude and direction.
- Understand the concept of dot product and its geometric interpretation.

3. Application of Vectors in Problem Solving:

- Solve vector-related problems in various contexts, such as physics, navigation, or engineering.
- Utilize vector concepts to analyze forces, displacements, or velocities in real-life scenarios.
- Apply vector operations to determine resultant vectors or find missing vector components.

4. Integration of Interdisciplinary Knowledge:

- Recognize the interdisciplinary nature of vectors by applying them in other subjects, such as physics, engineering, or art.
- Make connections between vector concepts learned in mathematics and their application in different fields.

5. Critical Thinking and Problem-Solving Skills:

- Develop the ability to analyze and interpret vector diagrams and representations.
- Apply logical reasoning to solve vector-related problems.
- Enhance critical thinking skills by identifying appropriate vector operations for specific situations.

ASSIGNMENT QUESTIONS:

1. Given two vectors A = 3i - 4j + 2k and B = 2i + j - 3k, calculate: a) The magnitude of vector A. b) The unit vector in the direction of vector B. c) The dot product of vectors A and B.
2. Find the resultant of the following vector addition: A = 4i + 3j - 2k, B = -2i + 5j + 4k, C = 3i - 2j + k.
3. Determine the angle between vectors A = 2i + j - 3k and B = 4i - j + 2k.
4. A car travels 60 km due north and then 40 km due west. Calculate the car's displacement vector from the starting point. a) Represent the displacement vector graphically. b) Determine the magnitude and direction of the displacement vector.
5. A force of 10 N acts in the direction of vector A = 3i + 2j - k. Calculate the work done by the force when it displaces an object through a distance of 5 meters in the direction of vector B = 2i - 3j + 4k.
6. Given vectors A = 2i + 3j - 4k and B = 3i - 5j + 6k, determine: a) The vector that represents the sum of A and B. b) The vector that represents the difference between A and B.
7. The position vector of a point P with respect to the origin O is given by r = 2i + 3j + 4k. Find the distance of point P from the origin.
8. In a triangle ABC, the position vectors of points A, B, and C with respect to the origin O are given by: OA = i + 2j - k, OB = 2i - j + k, OC = 3i - 4j + 2k. Determine the lengths of sides AB, BC, and AC.
9. A particle moves in a plane with a velocity vector v = 3i + 4j. After 5 seconds, its position vector with respect to the origin becomes r = 7i + 15j. Find the initial position vector of the particle.

Remedial measures:

1. Reinforce Basic Concepts: Make sure students have a solid understanding of fundamental mathematical concepts that form the basis of vectors, such as coordinate geometry, algebra, and trigonometry. Provide additional practice and explanations for these concepts if needed.
2. Provide Visual Aids: Use visual aids, diagrams, and interactive online tools to help students visualize vectors and their operations. Demonstrating vector addition, subtraction, and other operations graphically can enhance understanding and retention.

Lesson Plan: Three-Dimensional Geometry

No. of days: 15

Learning Objectives:

1. Understand the concepts of three-dimensional geometry, including points, lines, and planes in space.
2. Identify and describe the basic elements of three-dimensional figures, such as edges, vertices, and faces.
3. Apply the distance and section formulas to solve problems related to three-dimensional geometry.
4. Analyze and interpret real-life situations using three-dimensional geometry concepts.
5. Develop spatial reasoning skills and visualization abilities.

Previous Knowledge Testing:

1. Begin the lesson by asking students to recall and describe the basic elements of two-dimensional geometry, such as points, lines, and angles.
2. Pose questions related to coordinate geometry and the Cartesian coordinate system to assess students' understanding of the two-dimensional plane.
3. Conduct a quick review of the distance formula and its application in calculating the distance between two points in a plane.

Vocabulary Used:

1. Three-dimensional geometry
2. Points, lines, and planes
3. Edges, vertices, and faces
4. Distance formula
5. Section formula
6. Intersection
7. Parallel
8. Perpendicular

AID USED: Media links , ncert exampler ,Rd Sharma.

Interdisciplinary Linkage:

1. Science: Discuss the relevance of three-dimensional geometry in understanding molecular structures and crystallography.
2. Engineering: Explore the application of three-dimensional geometry in architectural design, civil engineering, and computer graphics.
3. Art: Discuss the use of perspective and three-dimensional representations in visual arts, such as sculpture and painting.

Art Integration:

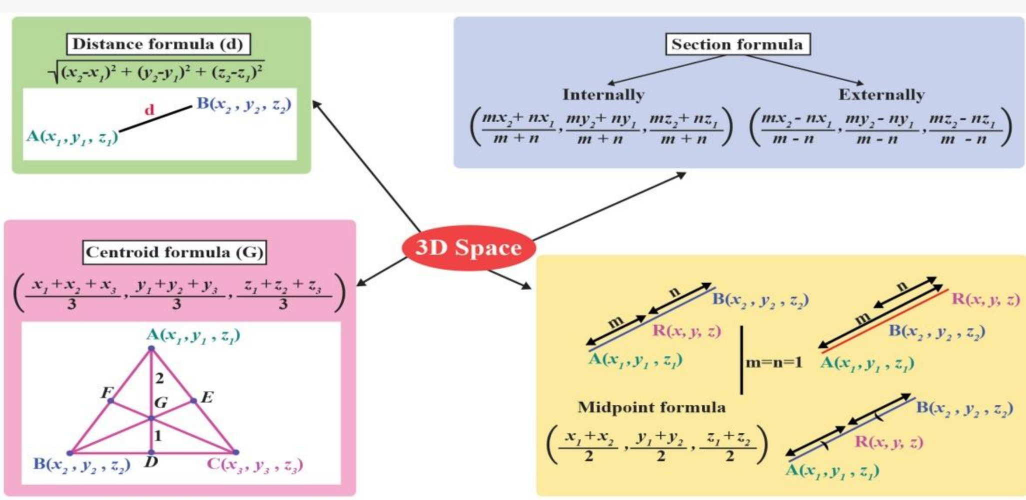
1. Ask students to create three-dimensional models of simple geometric figures using clay, paper, or other materials.
2. Encourage students to draw three-dimensional representations of everyday objects, landscapes, or abstract concepts using shading, perspective, and depth.

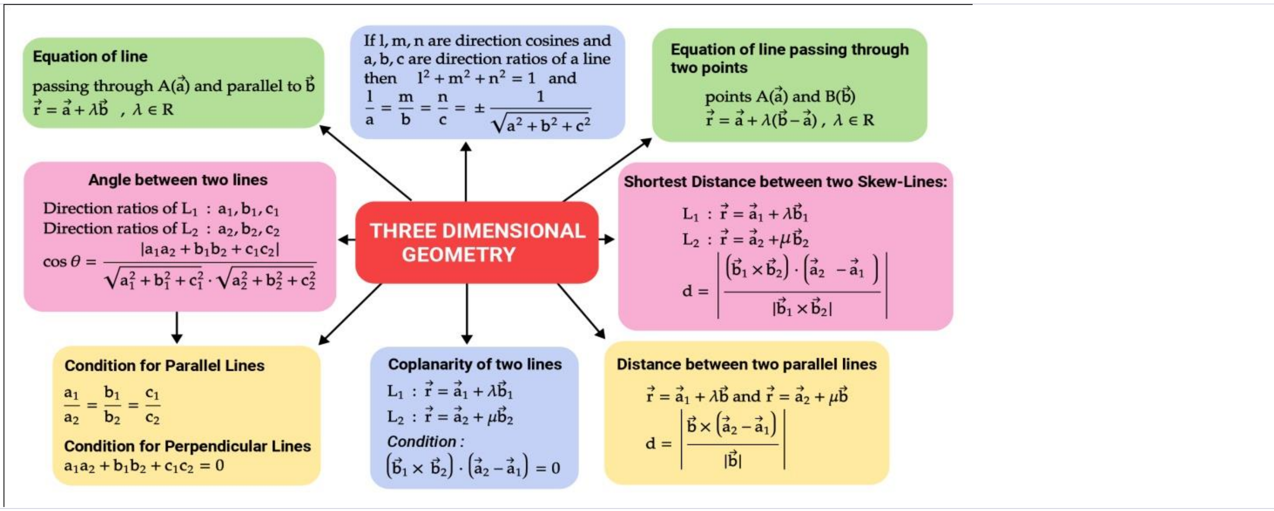
Experiential Learning:

1. Provide students with three-dimensional models or physical objects to explore and analyze, encouraging them to visualize and manipulate them.
2. Use interactive virtual simulations or educational apps to enhance students' understanding of three-dimensional concepts.
3. Conduct a virtual or real-life field trip to a local architectural site, sculpture gallery, or engineering firm to demonstrate practical applications of three-dimensional geometry.

Procedure:

1. Begin the lesson by reviewing two-dimensional geometry concepts, including points, lines, angles, and coordinate geometry.
2. Introduce the concept of three-dimensional geometry and its relevance in various fields.
3. Present examples of three-dimensional figures and their basic elements, emphasizing edges, vertices, and faces.
4. Demonstrate the distance formula in three-dimensional space and its application in finding the distance between two points.
5. Discuss the section formula and its usage in dividing line segments into a given ratio in three-dimensional space.
6. Provide examples and problem-solving exercises related to calculating distances, dividing line segments, and solving real-life problems using three-dimensional geometry concepts.
7. Engage students in hands-on activities, such as creating three-dimensional models or drawing three-dimensional representations, to reinforce their understanding.
8. Facilitate class discussions and encourage students to share their observations and reasoning behind solving problems related to three-dimensional geometry.
9. Use technology resources, including educational apps or virtual simulations, to enhance students' exploration and understanding of three-dimensional concepts.
10. Conclude the lesson by summarizing the key concepts and providing additional resources for further practice and exploration.





Inclusive Learning Practices:

1. Use visual aids, diagrams, and models to cater to different learning styles.
2. Provide additional support or scaffolding for students with learning

ASSIGNMENT Questions:

1. Find the coordinates of the midpoint of the line segment joining the points A(2, -1, 4) and B(5, 3, -2).
2. Determine the distance between the points P(1, -2, 3) and Q(-4, 5, 2).
3. Given the points A(2, 1, -3), B(-1, 3, 4), and C(5, -2, 0), determine whether the three points are collinear.
4. Find the ratio in which the line segment joining the points P(2, 4, -1) and Q(5, -1, 3) is divided by the xy-plane.
5. Determine the equation of the line passing through the points A(2, -1, 3), B(4, 5, -2),.

6. Determine the equation of the line parallel to the line, passing through the points A(1, -2, 3) and B(3, 4, -1).
7. Find the angle between two lines passing through the points (1,2,3) and (7,8,9)
8. Determine the distance between the parallel planes represented by the equations $2x + 3y - 4z = 5$ and $2x + 3y - 4z = 10$.

Remedial Measures for Three-Dimensional Geometry: for weak , average and gifted students

1. Review Basic Concepts: If students are struggling with three-dimensional geometry, it is essential to revisit and reinforce their understanding of basic concepts in two-dimensional geometry. Ensure that they have a solid foundation in points, lines, angles, and coordinate geometry before proceeding to three-dimensional concepts.
2. Visualizations and Manipulations: Three-dimensional geometry often requires spatial reasoning skills and the ability to visualize objects in space Encourage them to rotate, analyze, and identify different components of the figures.

3. Hands-on Activities: Engage students in hands-on activities that involve constructing three-dimensional models using different materials like clay, paper, or blocks. These activities can help students develop a deeper understanding of spatial relationships and the characteristics of three-dimensional figures.
4. Use Visual Aids: Utilize visual aids, diagrams, and illustrations to explain three-dimensional concepts. Use clear and labeled diagrams to represent figures, their components, and their relations.
5. Collaborative Learning: Foster collaborative learning environments where students can discuss and work together on three-dimensional geometry problems. Encourage them to explain their thought processes, discuss strategies, and learn from each other's approaches. Peer discussions and interactions can promote a deeper understanding of the subject matter.

6. Technology Integration: Utilize interactive software, virtual simulations, or educational apps that offer visualizations and interactive activities related to three-dimensional geometry. These tools can provide students with additional support and make abstract concepts more accessible.
7. Individualized Support Offer one-on-one sessions, tutorials, or additional resources tailored to their specific needs. Monitor their progress closely and provide constructive feedback to help them overcome challenges.
8. Regular Assessments: Conduct regular assessments, including formative and summative assessments, to gauge students' understanding of three-dimensional geometry. Identify areas of weakness and provide timely feedback to guide their learning and address any misconceptions.

Lesson Plan : Probability

No of days: 20

Learning Objectives:

1. Understand conditional probability and its application in real-life scenarios.
2. Apply the multiplication theorem to solve probability problems involving multiple events.
3. Identify and distinguish between independent and dependent events.
4. Understand the concept of total probability and its relevance in solving complex probability situations.
5. Apply Bayes' theorem to calculate probabilities in real-life scenarios.
6. Define random variables and analyze their properties.
7. Calculate the mean of a random variable using appropriate formulas.

Previous Knowledge Testing: To assess students' prior knowledge, begin the lesson by asking the following questions:

1. Define conditional probability and provide an example.
2. What is the multiplication theorem, and when is it used?
3. Explain the difference between independent and dependent events.
4. Have you encountered total probability and Bayes' theorem before? If yes, provide an example.

Interdisciplinary Linkage Activity: Introduce an interdisciplinary activity to connect probability with another subject, such as physics or computer science. For example, discuss how probability is used in quantum mechanics or in designing algorithms.

Art Integration: Encourage art integration by asking students to create visual representations of conditional probability or random variables using diagrams or graphs.

Vocabulary Used:

1. Conditional probability
2. Multiplication theorem
3. Independent events
4. Dependent events
5. Total probability

6. Bayes' theorem
7. Random variable
8. Mean of a random variable

Procedure:

1. Begin the lesson by revisiting the concept of probability and its basic principles.
2. Introduce conditional probability as the probability of an event occurring given that another event has already occurred.
3. Provide examples of real-life situations where conditional probability is relevant, such as medical tests or weather forecasting.
4. Explain the multiplication theorem and how it is used to calculate the probability of the intersection of multiple events.
5. Differentiate between independent and dependent events and provide examples for each.
6. Discuss the concept of total probability and how it can be applied to calculate the probability of an event using different scenarios.
7. Introduce Bayes' theorem as a method to update probabilities based on new information.
8. Illustrate Bayes' theorem with an example, such as a medical diagnosis or a courtroom scenario.
9. Define random variables and explain how they are used to represent uncertain quantities.
10. Discuss the mean of a random variable as the expected value or average outcome.
11. Explain how to calculate the mean of a discrete random variable using the appropriate formulas.
12. Engage students in solving practice problems and examples related to conditional probability, multiplication theorem, independent events, total probability, Bayes' theorem, random variables, and the mean of a random variable.
13. Encourage class discussions and group activities to deepen understanding and address any questions or misconceptions.

Formula

$$P(A) = \frac{\text{number of favourable events}}{\text{number of total events}}$$

$$P(A) = \frac{n(A)}{n}$$

$$P(B) = \frac{n(B)}{n}$$

$$P(A \cap B) = P(A)\,P(B)$$

for Mutually Exclusive Events

$$P(A \cup B) = P(A) + P(B)$$

for non-Mutual Events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

for Conditional probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Bayes' Theorem

Let $A_1, A_2, \ldots, A_k$ be a collection of k mutually exclusive and exhaustive events with *prior* probabilities $P(A_i)$ $(i = 1, \ldots, k)$. Then for any other event B for which $P(B) > 0$, the *posterior* probability of A_j given that B has occurred is

$$P(A_j \mid B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B \mid A_j)P(A_j)}{\sum_{i=1}^{k} P(B \mid A_i) \cdot P(A_i)} \qquad j = 1, \ldots, k \qquad (2.6)$$

Expected Value or Mean

$$E(X) = \mu = \sum_{i=1}^{n} x_i P(x_i)$$

$E(X)$ is the Expected Value

μ is the mean

x_i is an outcome

$P(x_i)$ is the probability of the outcome

Inclusive Learning:

1. Use visual aids, such as diagrams, graphs, or flowcharts, to support visual learners.
2. Provide clear explanations and step-by-step instructions to aid comprehension.
3. Offer additional support to struggling students through one-on-one assistance or differentiated assignments.
4. Encourage collaborative learning by assigning group tasks and promoting peer interaction.

Expected Learning Outcomes: By the end of this lesson, students should be able to:

1. Understand and apply conditional probability in various contexts.
2. Use the multiplication theorem to calculate probabilities of multiple events.
3. Differentiate between independent and dependent events.
4. Apply the concept of total probability to solve complex probability situations.
5. Apply Bayes' theorem to update probabilities based on new information.
6. Define random variables and analyze their properties.
7. Calculate the mean of a random variable using appropriate formulas.

Assignment:

Assign students a set of problems and exercises covering conditional probability, multiplication theorem, independent events, total probability, Bayes' theorem, random variables, and the mean of a random variable. Provide feedback and discuss the solutions in the following session.

Assignment: Multiple-Choice Questions on Probability

Instructions: Choose the correct option for each question. Circle the corresponding letter (A, B, C, or D) for your answer.

1. What is the probability of rolling an even number on a fair six-sided die?
   A. 1/6 B. 1/3 C. 1/2 D. 2/3
2. Two events A and B are said to be independent if:
   A. They cannot occur at the same time. B. The occurrence of one does not affect the probability of the other. C. They are mutually exclusive events. D. Their probabilities add up to 1.
3. The multiplication theorem is used to calculate the probability of:
   A. Independent events. B. Dependent events. C. Mutually exclusive events. D. Complementary events.
4. In a deck of 52 playing cards, what is the probability of drawing a red card?
   A. 1/13 B. 1/26 C. 1/2 D. 1/4
5. If P(A) = 0.4 and P(B) = 0.6, what is the probability of both events A and B occurring?
   A. 0.24 B. 0.6 C. 0.4 D. 0.1
6. The total probability theorem is used to calculate the probability of an event:
   A. When there are multiple possible outcomes. B. When there is only one possible outcome. C. When the event is certain to occur. D. When the event is impossible to occur.
7. Bayes' theorem is used to update the probability of an event based on:
   A. The prior probability and the conditional probability. B. The total probability and the multiplication theorem. C. The mean of a random variable and the standard deviation. D. The independence of two events.
8. A random variable represents:
   A. An uncertain quantity. B. A certain quantity. C. The sum of two independent events. D. The difference between two dependent events.

9. The mean of a random variable is also known as its:
A. Median B. Mode C. Variance D. Expected value.

Remedial Measures: for weak , average , gifted students

For students who require additional support, offer remedial measures such as:

1. Extra practice problems and worksheets with increasing difficulty levels.Provide step-by-step explanations and guided examples.
2. Conduct additional review sessions or one-on-one tutorials.
3. Encourage the use of online resources, educational videos, or interactive simulations to reinforce understanding.

LESSON PLAN: LINEAR PROGRAMMING PROBLEMS

No. of days : 5

Learning Objectives:

Understand the concept of linear programming and its applications.

1. Learn to formulate linear programming problems.
2. Apply graphical and algebraic methods to solve linear programming problems.
3. Analyze and interpret the results obtained from linear programming solutions.
4. Develop critical thinking and problem-solving skills.
5. Foster interdisciplinary connections through real-life applications.
6. Integrate art into mathematical concepts.
7. Promote experiential learning through hands-on activities.
8. Cultivate inclusive learning practices by incorporating different learning styles and abilities.

Previous Knowledge Testing:

1. Begin the lesson by asking students to recall and define key terms such as constraints, objective function, feasible region, and optimization.
2. Present a scenario-based problem and ask students to identify if it can be modeled as a linear programming problem.

Vocabulary Used:

1. Linear programming
2. Constraints
3. Objective function
4. Feasible region
5. Optimization
6. Inequality
7. Corner point

AIDS USED: Media links ,ncert ,Rd Sharma.

Interdisciplinary Linkage: Discuss real-life applications of linear programming in various fields such as:

1. Economics: Optimal production planning, resource allocation.
2. Business: Supply chain management, inventory control.
3. Environmental Science: Optimal land use planning, conservation strategies.
4. Engineering: Optimal design and allocation of resources.

5. Social Sciences: Allocation of funds and resources for public projects.

Art Integration:

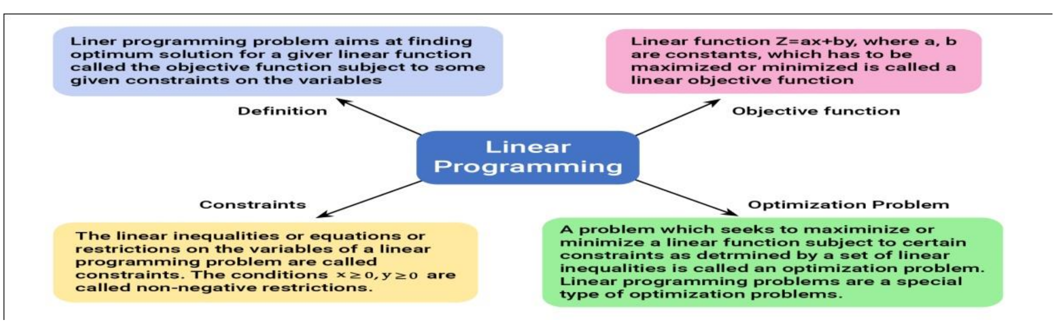
1. Ask students to create visual representations, such as graphs or diagrams, of linear programming problems.
2. Incorporate art projects that involve creating posters or infographics to illustrate real-life applications of linear programming.

Experiential Learning:

1. Divide students into small groups and provide them with real-world scenarios.
2. Have each group formulate a linear programming problem based on the scenario and solve it using graphical or algebraic methods.
3. Conduct a class discussion to analyze and compare the results obtained by different groups.

PROCEDURE :

1. Applying graphical methods to solve linear programming problems.
2. Analyzing and interpreting the graphical solutions.
3. Introducing the concept of corner points and their significance.



1) SOLVE: Max(Z)= 40X+30Y,

Subject to constraints:

2x+y < 6

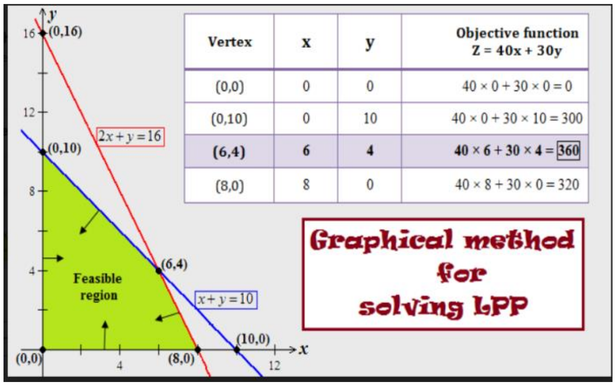
X+Y < 10

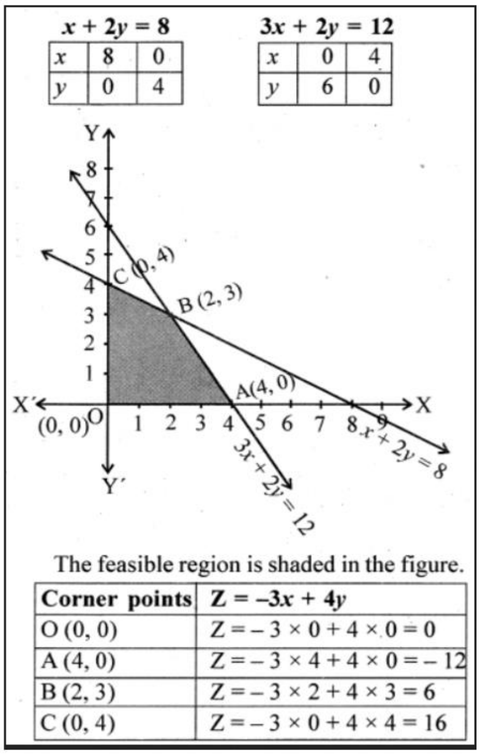
2) SOLVE: Max(z)= -3x +4y

Subject to constraints:

x+2y< 8

3x+2y<12





Inclusive Learning Practices:

1. Provide visual aids and diagrams to support visual learners.
2. Offer alternative problem-solving strategies to accommodate different learning styles.
3. Encourage peer collaboration and group discussions.
4. Provide additional support or differentiated instructions for students with learning difficulties.

Learning Outcomes: By the end of the lesson, students should be able to:

1. Formulate linear programming problems from real-life scenarios.
2. Solve linear programming problems using graphical and algebraic methods.
3. Interpret and analyze the solutions obtained from linear programming models.
4. Explain the significance of corner points and shadow prices.
5. Apply critical thinking and problem-solving skills to real-world situations.
6. Recognize interdisciplinary connections between linear programming and other fields.
7. Integrate art into mathematical concepts.
8. Engage in experiential learning through hands-on activities.

Remedial measures : for week students ,average and gifted students

1. Concept Reinforcement:
- Identify the specific areas where students are struggling or have misconceptions.

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2. Visual Representation:
- Utilize visual aids such as graphs, diagrams, and charts to help students better understand the graphical representation of linear programming problems.
- Provide step-by-step guidance on how to plot constraints and objective functions on the graph.
- Encourage students to create their own visual representations to solidify their understanding.
3. Guided Practice:
- Offer guided practice sessions where students can solve linear programming problems with the teacher's assistance
4. Collaborative Learning:
- Encourage students to work in pairs or small groups to solve linear programming problems.
5. Reinforcement Through Technology
- Recommend online resources, tutorials, or video lessons that offer alternative explanations and examples for self-paced learning.
6. Individualized Support:
- Provide one-on-one guidance, extra practice materials, or alternative explanations tailored to their individual requirements.
- Offer extra assistance outside of regular class hours to address their specific concerns.
7. **Ongoing Assessment and Feedback**:
- Regularly assess students' understanding through quizzes, assignments, or in-class activities.
- Provide timely feedback to students, highlighting their strengths and areas for improvement.
- Use formative assessments to identify gaps in knowledge and adjust instruction