

First Term Examination (26 September 2017)

Class XII

Sub - Mathematics

(Set-B)

Time 3hrs

M.M. 100

Note: i) All questions are compulsory.

ii) Q1 to Q4 carry 1 mark each.

iii) Q5 to Q12 carry 2 marks each.

iv) Q13 to Q23 carry 4 marks each.

v) Q24 to Q29 carry 6 mark each.

SECTION - A

Q1. If A is square matrix of order 2 and $A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write value of $|A|$

Q2. $f : Q \rightarrow Q$ given by $f(x) = 2x - 3$ for $x \in Q$ is bijective find f^{-1} .

Q3. * be a binary operation defined by $x * y = x^2 + y^2 = 5x + 7y$ find $2 * 4$

Q4. Write Principal value of $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$.

SECTION - B

Q5. If $f : A \rightarrow B$ be defined by $f = \{(1,5), (2,6), (3,6), (4,7)\}$ show that f is neither one-one nor onto.

Q6. Find value of x if $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

Q7. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ Show that $A^2 - 5A + 7I = 0$ Hence find A^{-1}

Q8. If $xy + y^2 = \tan x + y$, then find $\frac{dy}{dx}$

Q9. If $\log y = \tan^{-1} x$ then prove that $(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$

Q10. A balloon, which always remains spherical, has variable diameter $\frac{3}{2}(2x+3)$ determine, the rate of change of volume with respect to x .

Q11. Find the intervals in which function $f(x) = 2x^3 + 9x^2 + 12x + 20$ (i) increasing (ii) decreasing.

Q12. Find the maximum and minimum value if any for function $f(x) = |\sin 4x + 3|$ on R .

SECTION - C

Q13. Show that relation R on set R of real numbers, defined as $R = \{(a, b) : a \leq b^3\}$ is neither reflexive nor symmetric nor transitive.

Q14. If $f : R^+ \rightarrow [-9, \infty]$ given by $f(x) = 5x^2 + 6x - 9$ prove that f is invertible with $f^{-1}(y) = \frac{\sqrt{54 - 5y} - 3}{5}$

Q15. Define binary operation * on set $A = \{1, 2, 3, 4\}$ as $a * b = ab \pmod{5}$. Show that 1 is identity for * and all elements of set A are invertible.

Q16. Prove that $\frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} = \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3}$

Q17. Show that
$$\begin{bmatrix} b^2 + c^2 & ab & ac \\ ba & c^2 + a^2 & bc \\ ca & cb & a^2 + b^2 \end{bmatrix} = 4a^2b^2c^2$$

Q18. Express the matrix $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.

Q19. If $f(x) = \begin{cases} \frac{1 - \cos kx}{x \sin x}, & x \neq 0 \\ \frac{1}{2}, & x = 0 \end{cases}$ is continuous at $x=0$, find value of k.

Q20. Show that function $f(x) = |x - 3|$ for $x \in R$ is not differential at point $x = 3$

Q21. If $y = \sin(\log x)$ then show that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

Q22. Verify Roller's Theorem for the following function $f(x) = e^x \{\sin x - \cos x\}$ on $\left[\frac{\pi}{4}, 5\frac{\pi}{4}\right]$

Q23. Use differentials to approximate the cube root of 127

SECTION - D

Q24. Show that relation R in set Let $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation. Show that all elements of $\{1, 3, 5\}$ are related to each other and all the elements of $\{2, 4\}$ are related to each other. But no element of $\{1, 3, 5\}$ is related to any element of $\{2, 4\}$.

Q25. Let $A = Q \times Q$ and let * be binary operation on A defined by $(a, b) * (c, d) = (ac, b+ad)$ for $(a, b), (c, d) \in A$. Determine whether * is commutative and associative. Then with respect to * on A

- i) Find the identity element in A
- ii) Find the invertible elements of A

Q26. Find the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of eq.

$$x - y + z = 4, \quad x - 2y - 2z = 9, \quad 2x + y + 3z = 1$$

Q27. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs. 300 and that on a chain is Rs. 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an LPP and solve it graphically.

Q28. Find the equation of normals to curve $3x^2 - y^2 = 8$ which are parallel to line $x + 3y = 4$

Q29. Show that the height of a closed right circular cylinder of given surface and maximum volume is equal to the diameter.