First Term Examination (26 September 2017)

Class XII

Sub - Mathematics

(Set-A)

Time 3hrs **M.M. 100**

Note: i) All questions are compulsory.
ii) Q1 to Q4 carry 1 mark each.
iii) Q5 to Q12 carry 2 marks each.
iv) Q13 to Q23 carry 4 marks each.
v) Q24 to Q29 carry 6 mark each.

SECTION - A

Q1. If A is an invertible matrix of order 2 and (A)=4 then write value $\det(A^{-1})$

Q2. Show that function $f: N \to N$ given $f(x) = 2x$ is not onto.

Q3. * be a binary operation on set Q of rational numbers defined as a * b = a + ab. Find 3 * 4.

Q4. Find the value of $Sin^{-1}\left(\sin\frac{4\pi}{5}\right)$.

SECTION – B

Q5. Show the function $f: A \to B$ which is defined by $f = \{(1,5),(2,6),(3,7),(4,8)\}$ is invertible function.

Q6. If $\begin{bmatrix} 3 & 4 \\ 2 & x \end{bmatrix}\begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix}$ find value of x.

Q7. If A is square matrix such that $|A| = 5$ write value of $|AA^T|$.

Q8. If xy = sec (x + y) then find $\frac{dy}{dx}$

Q9. If $y = Sin^{-1}x$, then prove that $(1-x)^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

Q10. Find rate of change of area of circular disc with respect to its circumference when radius is 3cm.

Q11. Find the intervals in which function $f(x) = 2x^3 - 9x^2 + 12 + 15$ (i) increasing (ii) decreasing.

Q12. Find the maximum and minimum values if any for function $f(x) = -|x+1| + 3\, on\, R$.

SECTION – C

Q13. Show that relation R on set R of real numbers, defined as $R=\{(a,b:a\le b^2)\}$ is neither reflexive nor symmetric nor transitive.

Q14. Let $f:R_+\to[-5,\infty)$ be function defined as $f(x)=9x^2+6x-5$ show that f is invertible and $f^{-1}(y)=\left(\frac{\sqrt{y+6}-1}{3}\right)$

Q15. Define a binary operation * on set $A=\{0,1,2,3,4,5,\}$ as a*b = a + b (mod 6). Show that zero is the identity for this operation and each element 'a' of set is invertible with (6 – a) being inverse of a.

Q16. $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

Q17. Show that $\begin{bmatrix}(b+c)^2 & ba & ca\\ ab & (c+a)^2 & cb\\ ac & bc & (a+b)^2\end{bmatrix}=2abc\left[a+b+c\right]^3$

Q18. Express the matrix $A=\begin{bmatrix}4 & 2 & -1\\ 3 & 5 & 7\\ 1 & -2 & 1\end{bmatrix}$ as sum of symmetric and skew symmetric matrix

Q19. Find values of K so that function f is continuous at indicated point at $x=\frac{\pi}{2}$

$$f(x)=\begin{cases}\frac{K\cos x}{\pi-2x}, & if\ x\ne\frac{\pi}{2}\\ 3 & if\ x=\frac{\pi}{2}\end{cases}$$

Q20. Show that function $f(x)=|x-1|$ for $x\in R$ is not differential at point $x=1$

Q21. If $y=(\tan^{-1}x)^2$ then show that $(x^2+1)^2\frac{d^2y}{dx^2}+2x(x^2+1)\frac{dy}{dx}=2$

Q22. Verify Lagrange's mean value Theorem for function $f(x)=2\sin x+\sin 2x\ on\left[0,\pi\right]$

Q23. Using differentials, find the approximate value of $\sqrt{401}$

SECTION - D

Q24. Let $A=\{1,2,3....9\}$ and R be relation in A × A defined in A × A defined by (a,b) R (c, d) if a + d = b+c for (a,b) (c,d) in a×A. prove that R in equivalence relation. Also obtain equivalence class $\left[2,5\right]$

Q25. Let A=N×N and * be binary operation on A defined by (a, b) * (c, d)=(a+c, b+d). Show that * is commutative and associative. Also find the identity element for * on A if any

Q26. If $A=\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & 2 \end{bmatrix}$ find A^{-1} Hence using A^{-1} solve the system of eq.

$2x-3y+5z=11,\ 3x+2y-4z=-5,\ x+y-2z=-3$

Q27. A factory makes two types of items A and B, made of plywood. One piece of item A requires 5 minutes for cutting and 10 minutes for assembling. One piece of item B requires 8 minutes for cutting and 8 minutes for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours for assembling. The profit on one piece of item A is Rs. 5 and that on item B is Rs. 6. How many pieces of each type should the factory make so as to maximize profit? Make it as an LPP and solve it graphically.

Q28. Find the equation of normals to curve $y=x^3+2x+6$ which is parallel to line $x+14y+4=0$

Q29. Show that the right circular cylinder open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.