Maths

Chapter name: quadratic equations

Time allotted for lesson

This lesson is divided across three modules. It will be completed in three class meetings

Prerequisite knowledge

Polynomials

Short description of the lesson

This lesson will introduce learners to quadratic equations and the different methods to find their roots. They will also learn to drive the quadratic formula and use it to find the roots of a quadratic equation. Further, they will learn to find the nature of the roots of a quadratic equation as well as the sum and the product of the roots.

Objectives

- Check if a given equation is a quadratic equation
- Given situation in the form of a quadratic equation
- Find the roots of a quadratic equation by factorisation.

- Derive the quadratic formula
- the solution of a quadratic equation using the quadratic formula
- Find the nature of the roots of a quadratic equation.
- Find the sum and the product of the roots of a quadratic equation.

Aids

- Audio visual Aids
- Relevant modules from extra marks.
- https://youtu.be/mKV_wuZSlxI
- **https://youtu.be/VTQSGYnqw1Y**
- **https://youtu.be/DLjq7v5506w**

Procedure

Teacher- student activities

A. Warm up session

Being the session by talking briefly about the history of a quadratic equations.

Then, ask a few simple questions to help students recall their prior knowledge about polynomials. You may write linear, quadratic and cubic

polynomials on the board and ask the students to identify the type of polynomial.

Explain that a quadratic equation is obtained when a quadratic polynomial is equated to zero.

Also, ask the students to recall the standard form of a quadratic polynomial and then show them the standard form of a quadratic equation .

You can also randomly arrange the terms of an equation and ask the students to arrange these terms in the descending order of their degrees and check if the equation is a quadratic equation or not .

At this point discuss the few situations where quadratic equations are used .

B. Chit activity

In this activity, students need to check if a given equation is a quadratic equation.

Teacher's notes

Divide the class into a few groups. Create chits with equations written on them. Present the chits to a group and ask a student from the group to pick up a chit. Thereafter, ask the student to write the equation on the board and simplify it. Once the equation is simplified, ask the student to identify if the equation is linear, quadratic or cubic. In the same manner, pass the chits to the other groups. The group that gets maximum write answers will be the winner.

Once the activity is done, present a few situations to the students and ask them to represent these situations in the form of a quadratic equation. You can write down the situation on the board and present the students with four options as possible answers (four equations) close. They need to identify the correct equation for the given situation.

C. Finding roots of quadratic equations

In this activity, students need to find the roots of a quadratic equations using the methods of factorisation.

Solving quadratic equations by factorising.

Direct instruction

1. Ask learners to solve the following six equations. As they work, walk around the classroom and assist where necessary. Once learners have had the chance to complete the questions, correct them in full on the board. Point out issues that you noticed while assisting learners as they were walking.
    1. (2 X – 1) (X + 3) = 0
    2. Y(3y+4) = 0
    3. $5 x^2 = 8x$
    4. 2a(a-1) +3(a-1) = 0
    5. $2x^2+3x = 27$
    6. $2(x-2) (x+2) + 6x = (x-1)^2$

    Solutions and notes

    1. (2x-1) (x+3) = 0

        2x-1 = 0 or x+3 =0

        2x = 1 x = -3

        x = ½

    2. y(3y+4) = 0

        y = 0 or 3y+4 = 0

$3y = -4$

$Y = -4/3$

Both the equations are already factorised and equal to zero and are therefore ready to be solved.

Many learners make the mistake of multiplying out first and trying to factorise again (and sadly make mistakes at this stage).

3. $5x^2 - 8x = 0$

   $x(5x-8) = 0$

   $x = 0$ or $5x-8 = 0$

   $5x = 8$

   $x = 8/5$

On one side and equal to zero before factorising.

4. $2a(a-1) + 3(a-1) = 0$

   $(a-1)(2a+3) = 0$

   $a-1 = 0$ or $2a+3 = 0$

   $a-1 = 0$ $\quad 2a = -3$

$a = -3/2$

Learners may need to be reminded how to factorise by grouping. There are two terms here with a common factor of a -1.

5. $2x^2+3x = 27$

   $2x^2 + 3x – 27 = 0$

   $(2x×9)\ (×-3) = 0$

   $2x +9 = 0$ or $x-3 = 0$

   $2x = -9$ $x+3 = 3$

   $× = -9/2$

Remind learners that when solving a quadratic equation, all terms need to be on one side and equal to zero before factorising.

$2(x-2)\ (x+2) +6x = (x-1)^2$

$2(x^2-4) +6x = x^2 -2x +1$

$2x^2 -8 +6x = x^2 -2x + 1$

$x^2 + 8x – 9 = 0$

$(x+9)\ (x-1) = 0$

$x+9 = 0$ or $x-1 = 0$

$x = -9$ $x = 1$

This question requires multiplying out, collecting like terms and then getting all the terms on one side before factorising.

2. Ask learners if they have any questions. Give learners and exercise to complete on their own.
3. Once learners have completed the exercise and it has been corrected, move on the next concept.

Solving quadratic equations using the quadratic formula

9. Tell learners that they will first practice solving a few equations using the quadratic formula before you show them how it was derived.

Learners should write the examples in their books and make notes as they do so.

Example

1. $x^2 + 6x - 2 = 0$

   $a = 1$ $b = 6$ $c = -2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(-2)}}{2(1)}$

$x = -3 \pm \sqrt{11}$

x = 0.32 or x = -6.32

Teaching notes

Say: write the values for A, write the values for a, b and c then use them to substitute into the quadratic formula.

Say: If the instruction in a question only says 'solve' then this answer is acceptable. This answer is also in simplest surd form. Should the instruction be to give your answer to two decimal places, then one more step on the calculator is required.

2. $5x^2 - 1/4x = 3$

   $5x^2 - 1/4x - 3 = 0$

   a =5 b = -1/4 c = -3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-\frac{1}{4} \pm \sqrt{(\frac{1}{4})^2 - 4(5)(-3)}}{2(5)}$$

x = ¾ or x = 4/5

Ask: Is this in standard form?

(This needs to be rewritten before listing the values of a, b and c).

Learners could multiply all terms by 4 (the LCD) if they wanted to but point out to them that they would get the same answers

From this step, the equation will be solved the same as above.

3. 7(x -3)(x +2) = 6x -2

   7(x^2 – x -6) = 6x -2

   7x^2 – 7x – 42 = 6x -2

   7x^2 – 13x – 40 = 0

   a = 7 b = -13 c = -40

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   $$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(7)(-40)}}{2(7)}$$

$x = 3.49$ or $x = -1.64$

Ask: Is this in standard form?

(Each side needs to be multiplied out and simplified then all terms need to be on one side equal to zero before listing the values of a, b and c).

From this step the equation will be solved the same as above.

E. Nature, Sum and Product of Roots

In this activity, students need to determine the nature of the roots of quadratic equations. They also need to find the sum and the product of the roots, if the roots exists for a given equation.

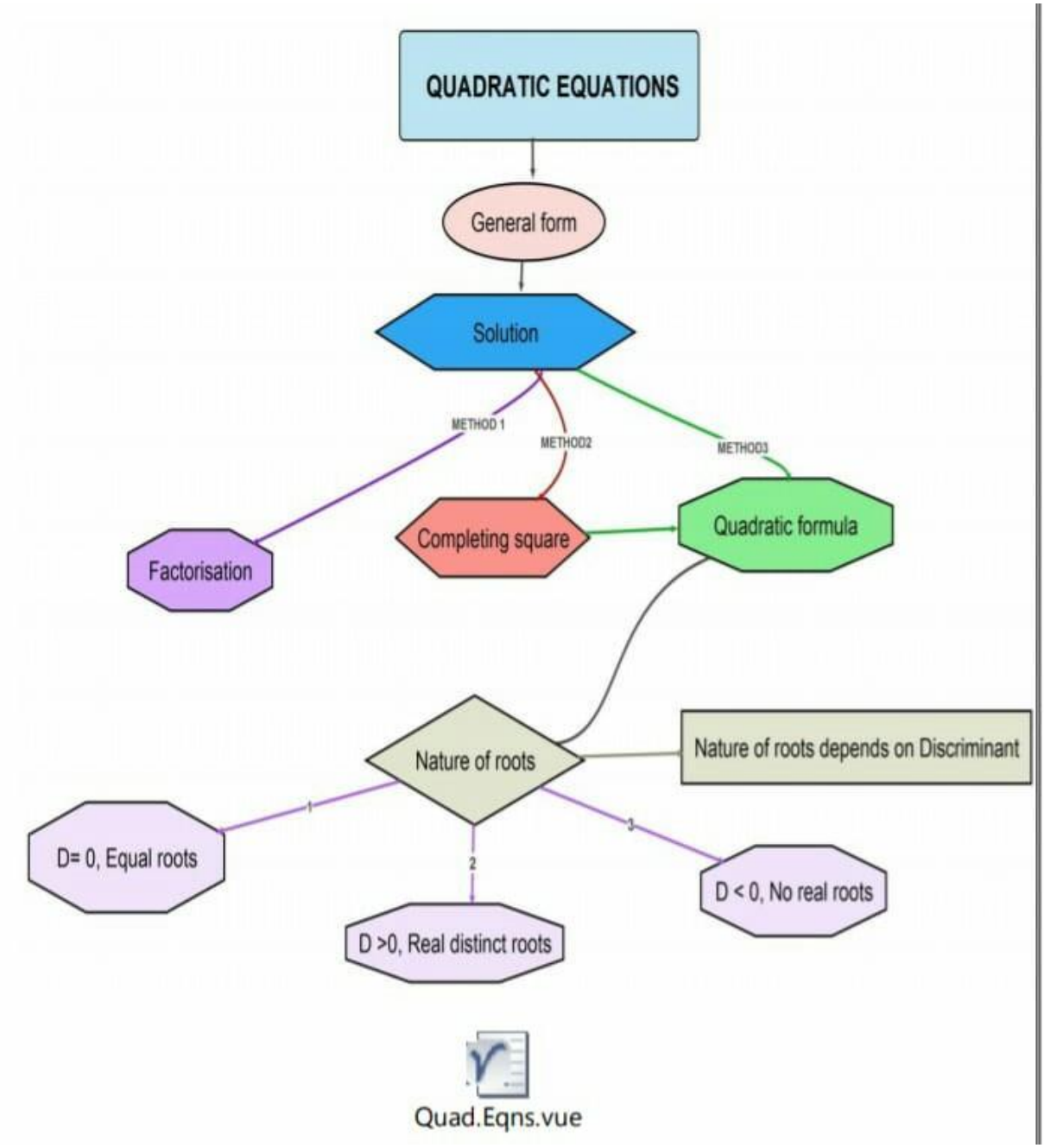
Teacher's Notes

Write a quadratic equation on the board and ask a student to find the discriminate of the equation. Once the discriminate is found, discuss the nature of the roots of the equation. If the equation has real roots, ask the student to find the roots and then find the sum and the product of the roots.

Then, you can present several other quadratic equations to the students and ask them to find the nature of the roots and the sum and the product of the roots, if the roots are real. The students can solve the equations in their notebooks.

Recapitulation

Recapitulation of the topic will be done through concept mapping.



Supplemental Activities

Ask the students to research on the use of quadratic equations.

Art integration

In this children will be taught to plot points on graph, they will learn to join points to form a parabolic shapes.

Expected Outcome

After studying this lesson, learners should be able to identify a quadratic equation.

They should also be able to find the roots of quadratic equations using the methods of factorisation and completing the ². They should also be able to drive the quadratic formula and use it to find the roots of a quadratic equation.

Further, they should be able to find the nature of the roots of a quadratic equation and the sum and the product of the roots.

Resources

NCERT book and extra marks.

Quadratic polynomial

Class room activity

Aim/objectives

- To draw the graph of a quadratic polynomial.
- To recognize the shape of the curve based on the sign of coefficient of x^2.
- To determine the number of zeroes(roots).

Key - concept

- Quadratic polynomial: An algebraic expression of the form $a x^2 + bx + c$, where a, b, c are real numbers and a is not equal to zero is called a quadratic polynomial in the variable x.
- Zeros of a quadratic polynomial: Let p (x) equal to $X^2 + bx + c$, a not equal to zero be a quadratic polynomial, then a real number 'A' is called as zero if and only if p (a) = 0.

Procedure

- Take a chart paper and paste a graph paper on it with the help of glue.
- Now, take a quadratic polynomial $q(x) = a x^2 + bx + c$, a not equal to zero.
- Now, we will we will take different values of a, b and c, by considering two cases:

  Case (1): $a > 0$ (+ve)

  case (2): $a < 0$ (-ve)

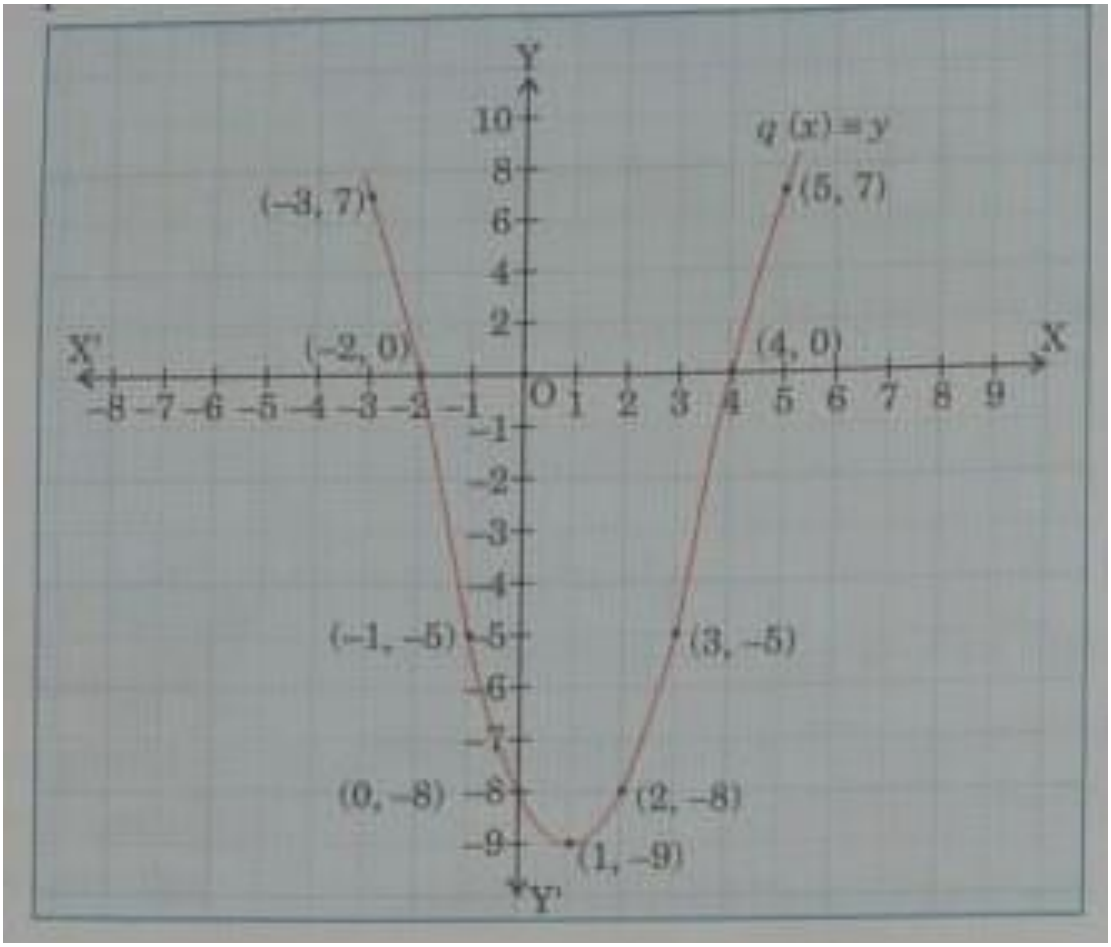
For a >0: let a=1, b=-2, c=-8

So, $q(x)=x^2 - 2x-8$

The following table gives the values of q(x) for various values of x.

x	-3	-2	-1	0	1	2	3	4	5
q(x)	7	0	-5	-8	-9	-8	-5	0	7

5. Now , plot these points on a graph paper and draw its graph as shown below:

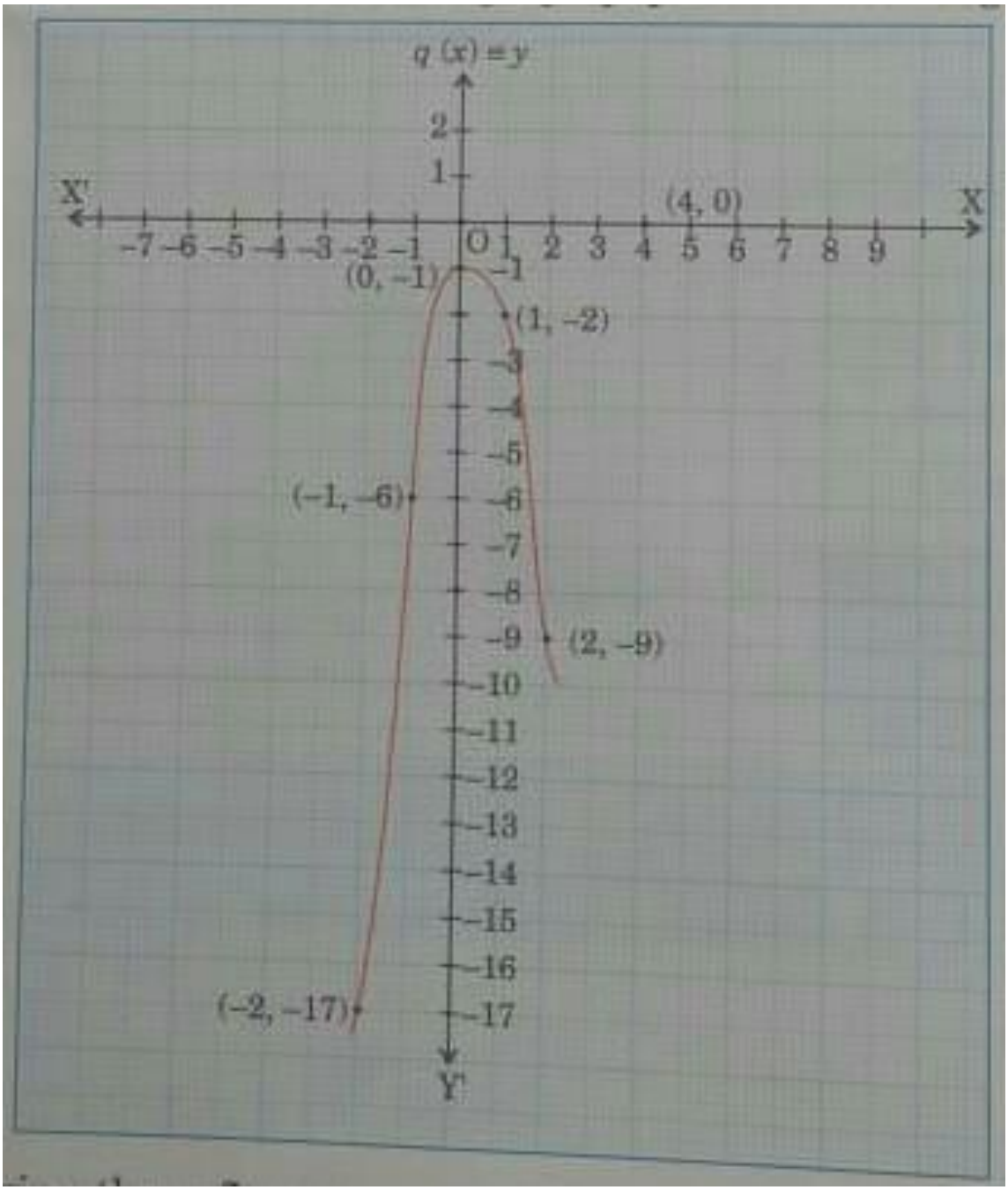


6. For a<0:let a = -3 ,b=2 ,c=-1.so , q(x) =-3x^2 +2x-1.

The following table gives the values of q(x) for various values of x.

x	0	1	-1	2	-2
q(x)	-1	-2	-6	-9	-17

Now, plot these points also on another graph paper and draw its graph as shown below



Now, by observing these figures we note down the number of times that these graph intersect with x – axis.

Observation

Graph -1 obtained in step 5 represents the parabola which opens upward because of +ve sign of coefficient of x^2.

Graph - 2 obtained in step 7 represents the parabola which opens downward because of -ve sign of coefficient of x^2.

Parabola given in the graph in step 5 cuts x-axis at two points. So, number of zeros of

$q(x) = x^2 – 2x - 8$ is two.

Parabola given in the graph in step 7 does not cuts x-axis. So, number of real zeros is 0.

Conclusion

The shape of the quadratic polynomial will always be a parabola.

If coefficient of x^2 is positive, then parabola opens upward and coefficients of x^2 is negative then parabola opens downward.

Number of zeros of quadratic polynomial = Number of times, points cuts x-axis

Application

The activity will be helpful to understand the geometrical representation of quadratic polynomial.

It will be helpful to find the number of zeros of quadratic polynomial using graph.

It will be helpful to identify the shape of quadratic polynomial just by observing the coefficient of x^2.

Assessment

Assessment questions will be given to solve.

Graded Exercises

Level -1

1. check whether x = (-1) is a solution of equation 4x-3x-1=0 (1 mark)

2. Find k , if one root of equation $x^2 + kx - 4 = 0$ (1 mark)

3. Solve by factorization: $9x^2 - 3x - 20 = 0$ (2 marks)

4. Solve by completing square method: $6x^2 - 13x - 5$ (2 marks)

5. Find the discriminant of the equation: $2x^2 - 7x + 3 = 0$ (1 mark)

6. Find the nature of roots of equation $9x^2+12x+4=0$ (1 mark)

7. Find k, if $2kx^2+6x+5=0$ has equal roots. (2 marks)

8. Solve : $\frac{x-1}{x}$ =3 (x≠0) (2marks)

9. The sum of roots of the equation $2x^2+7x-4$ (1mark)

10. The product of roots of equation $2x^2+7x-4$ (1mark)

11. If 2 is a root of the equation $x^2-bx+12=0$, find the value of 'b' and find the other root. (2marks)

Level-2

1. Find the value of k, for which the quadratic equation $(k-12)x^2+2(k-12)x+2=0$ has equal roots. (2marks)
2. Find the roots of the equation : (i)$4x^2+4\sqrt{3}x+3=0$ (2marks)

   (ii) $4\sqrt{3}x^2+5x-2\sqrt{3}=0$ (2marks)
3. The product of two consecutive integers is 182. Find the integers. (3marks)
4. Solve by factorization: $3x^2-2\sqrt{6x}+2$(2marks)
5. Find the roots of the equation $3x^2+6x+1=0$ by method of completing perfect square. (3marks)

6. If (-5) is a root of the quadratic equation $2x^2$ px-15=0 and the quadratic equation p (x^2 +x) +k=0 has equal roots, find the value of p and k. (3marks)
7. Find the value of p so that the equation px(x-3) +9=0 has equal roots. (2 marks)
8. Find the roots of the following equation:

$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$ (x≠-4.7)(3marks)

Level-3

1. Solve : $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ (a≠0, b≠0 ,x ≠0) (3marks)
2. Using quadratic formula, solve:
   i. $3a^2x^2+8abx+ab^2=0$(3marks)
   ii. $9x^2-3(a+b)\,x+ab=0$(3marks)
   iii. $p^2x^2+(p^2-q^2)\,x-q^2=0$ (3marks)